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The Impact of Project-based Learning on Students' Collaborative and Mathematical Communication Competencies: A PLS-SEM Analysis

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This study evaluates the effectiveness of Project-Based Learning (PjBL) in developing collaborative competence (COC) and mathematical communication competence (MCC). Utilizing a diverse sample (N = 1,089) comprising 835 high school students (grades 10–12) and 254 mathematics education majors from various pedagogical universities in Vietnam, the research employs PLS-SEM to analyze the structural relationships between project design, teacher support, and the learning environment. The results confirm a high model fit (SRMR = 0.027; NFI = 0.966) and robust predictive power, with R2 values of 0.820 for COC and 0.701 for MCC. Teacher support emerged as the decisive mediator, while PLSpredict and CVPAT analyses validated the model's superior out-of-sample predictive accuracy (Q2predict > 0.35) over traditional linear models. These findings provide a rigorous empirical foundation for standardizing PjBL, emphasizing that well-structured designs and proactive guidance are essential for fostering 21st-century skills across both secondary and teacher-training contexts in Vietnamese mathematics education.

Keywords: project-based learning, mathematical communication competence, collaborative competence, PLS-SEM analysis

INTRODUCTION

Current global educational reforms prioritize the development of comprehensive student competencies. Among these, collaborative competence and mathematical communication competence are pivotal for navigating modern society, where interaction and teamwork directly influence academic success and future professional trajectories. These competencies enable students to function effectively within collaborative learning environments, particularly when addressing complex, multifaceted problems. While collaborative competence fosters social skills and knowledge sharing through group dynamics, mathematical communication competence serves a dual purpose. According to Ningsih et al. (2025), it allows learners to logically articulate and justify mathematical reasoning using diverse representational tools, while simultaneously acting as a medium for the co-construction of knowledge. By fostering multidimensional understanding through authentic problem-solving, these

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interconnected competencies form the foundation for student-centered, interactive education.

As educational paradigms in Vietnam and globally shift from content-based to competency-based assessment, fostering collaboration (COC) and mathematical communication (MCC) has become an essential priority. This transition demands innovative pedagogical strategies that move beyond traditional instruction to promote comprehensive student development. Project-Based Learning (PjBL) is a high-potential method in this context, offering a realistic environment where students can actively construct knowledge through group work (Rehman et al., 2023). In mathematics specifically, PjBL enables students to solve complex problems and articulate abstract concepts, providing a dual-action mechanism for the simultaneous development of both MCC and COC skills.

Despite extensive research on Project-Based Learning (PjBL), there remains a scholarly gap regarding its specific impact on collaboration and mathematical communication competencies at the secondary level. To address this, the present study analyzes critical factors—including project design, teacher support, and the learning environment—to predict how PjBL fosters these dual competencies. In this context, collaboration encompasses group dynamics, shared responsibility, and progress management, which are essential 21st-century skills for solving real-world problems. Craig and Marshall (2019) argue that PjBL provides an optimal environment for these skills to flourish, leading to measurable improvements in student performance. Simultaneously, mathematical communication serves as a vehicle for students to articulate ideas, organize thinking, and critically evaluate problem-solving strategies (Holmes & Hwang, 2016). This shift from knowledge transmission to competency-based learning allows students to debate and construct mathematical knowledge interactively. Furthermore, Rehman et al. (2023) highlight that PjBL in mathematics significantly enhances not only teamwork but also critical thinking. By investigating the simultaneous development of these competencies, this study aligns with modern educational requirements, offering teachers a practical framework for designing interactive, student-centered mathematics instruction.

Recent findings by Ningsih et al. (2025) demonstrate that Project-Based Learning (PjBL) outperforms traditional methods in fostering students' communication and collaboration. By engaging in the PjBL cycle—ranging from group task allocation to peer feedback and product presentation—students simultaneously enhance their collaboration competency (COC) and mathematical communication competency (MCC). Furthermore, Zhang et al. (2023) highlighted that the quality of group interaction accounts for approximately 50% of PjBL's overall effectiveness. Building on this foundation, the present study analyzes and predicts the factors influencing the development of COC and MCC within the PjBL framework. It specifically aims to evaluate PjBL's efficacy and propose standardized implementation strategies tailored to the Vietnamese educational context. The findings offer significant implications: providing teachers with a roadmap for designing competency-based activities and offering administrators empirical evidence to refine teacher training and educational

policy. Ultimately, this research contributes to the global discourse on competency-based mathematics education through the lens of local practice.

Literature Review

PjBL: Theoretical Foundations and Core Mechanisms

PjBL is grounded in John Dewey's "learning by doing" and the constructivist paradigm, where knowledge is viewed as a socially situated construction rather than passive absorption (Ningsih et al., 2025; Ferrero et al., 2021). Unlike traditional rote memorization, PjBL positions students as autonomous "researchers" who navigate an interacting cycle: starting with a "driving question," moving through independent inquiry, and culminating in tangible, public artifacts (Markula & Aksela, 2022; Li et al., 2025).

The efficacy of this model lies in the transfer of learning responsibility (Stehling & Munzert, 2018). While PjBL is teacher-directed in its design, it is inherently learner-led in execution, transforming abstract disciplinary concepts into presentable products through four core stages: project design, persistent inquiry, in-depth modeling, and public presentation (Chen & Yang, 2019; Zhang et al., 2024).

Empirical Evidence, Global Trends and Identified Gaps

Recent meta-analyses of over 100 empirical studies confirm that PjBL significantly enhances cognitive, affective, and behavioral outcomes in STEM and higher education (Loyens et al., 2023; Cruz et al., 2022). A prominent global trend shows a research shift toward Asia, where PjBL interventions often yield more pronounced impacts on academic achievement than in Western contexts (Zhang et al., 2023). Beyond content mastery, PjBL fosters 21st-century competencies—such as creativity and self-efficacy—with medium-to-large effect sizes, facilitating a transition toward lifelong learning in applied mathematics (Zhang et al., 2024; Guo et al., 2020). Despite these documented benefits, several critical gaps necessitate further investigation. First, a mechanism-process gap exists, as current research emphasizes "before-and-after" results rather than providing structured modeling to explain how specific implementation mechanisms correlate with outcome stability (Navarro et al., 2025; Ibrahim, 2025). Second, a disciplinary and longitudinal gap remains regarding how PjBL sustains complex competencies, such as mathematical reasoning, over time within the unique socio-cultural landscapes of Southeast Asia (Hussein, 2021; Zhang et al., 2023). Finally, a conceptual-integration gap persists due to insufficient data on how the transition from "internal group inquiry" to "public presentation" specifically triggers the shift from general social collaboration to specialized academic discourse (Sfard, 2008; Zhang et al., 2024). By examining the synergy between teacher guidance and learner autonomy across the four core stages of PjBL implementation, this study seeks to bridge these empirical and theoretical voids.

Collaborative competence and mathematical communication competence

Collaborative Competence in Mathematics Education

In mathematics education, Collaborative Competence is a sophisticated socio-cognitive process essential for conceptual mastery, defined as a coordinated effort to build and

maintain a shared conception of mathematical tasks (Roschelle & Teasley, 1995; Cao, 2024). Unlike passive learning, it acts as a catalyst for the co-construction of knowledge through social negotiation, where modern frameworks like PjBL and RME foster "productive struggle" and collective argumentation (Mueller et al., 2014; Junaid, 2025). Structurally, this competence integrates four core dimensions: mathematical communication, which involves transcoding internal ideas into shared representations (NCTM, 2020); shared responsibility, rooted in "positive interdependence" and collective accountability (Johnson & Johnson, 2009; Muhamad Fadzil & Osman, 2025); constructive conflict resolution, which drives cognitive equilibration through the navigation of differing perspectives (Piaget, 1952); and mutual scaffolding, a reciprocal support process within the Zone of Proximal Development that allows groups to transcend individual limitations (Vygotsky, 1978). From a social-constructivist perspective, as students defend their reasoning against peer critique, they engage in rigorous metacognitive regulation, ultimately forming a robust mathematical identity (Dela Peña & Doronio, 2025). Consequently, collaborative competence serves as the cornerstone of 21st-century learning, bridging logical reasoning with social intelligence to prepare resilient innovators for an increasingly complex educational landscape.

Mathematical communication competence

Historically viewed as purely computational, modern mathematics education has undergone a paradigm shift that redefines mathematical communication as a core competence. Rocha and Babo (2024) define this competence as the ability to employ specialized mathematical language to convey, share, and decode information within problem-solving contexts. Beyond the mere presentation of symbols, it manifests as a readiness to establish shared understanding through written, spoken, and technological media (Niss & Højgaard, 2019). This complex social interaction requires mastering the "mathematics register"—a specialized system of terminology and expression—to accurately decode others' ideas while systematically presenting one's own (Boesen et al., 2014; Morgan et al., 2014). Contemporary research further delineates this competence into two interrelated dimensions: the receptive, involving the analysis of others' arguments, and the constructive, emphasizing the proactive presentation of ideas across varying technical depths (Aguilar & Castaneda, 2021; Niss & Højgaard, 2019). Supporting this, Sfard (2008) argues that mathematical thinking is inherently a discursive activity, equating the development of communication with the growth of mathematical thought itself. Moreover, drawing on Habermas's communicative theories, this competence involves participating in collective discourse, evaluating evidence, and justifying conclusions (Forster & Taylor, 2003). Such higher-order cognitive operations—including generalization and the refutation of unsupported arguments—not only deepen individual understanding but also empower students as responsible citizens capable of making data-driven decisions in society (Aguilar & Castaneda, 2021).

Collaborative vs. Mathematical Communication Competence

To analyze the impact of PjBL, it is critical to distinguish between two often-conflated social-cognitive competencies. (1) Collaborative Competence: Focuses on the interpersonal process. It involves the ability to work towards common goals through

shared responsibility, conflict resolution, and mutual support within the Zone of Proximal Development (ZPD) (Piaget, 1970; Ningsih et al., 2025). It is the "social engine" that allows learners to overcome individual cognitive limits; (2) Mathematical Communication Competence: Focuses on the discursive mastery of the discipline. It is the ability to use the "mathematics register"—specialized terminology, symbols, and gestures—to decode and encode mathematical ideas (Rocha & Babo, 2024; Boesen et al., 2014). Following Sfard (2008), this is not just "talking about math" but is synonymous with mathematical thinking itself.

Based on these identified gaps, the study proposes a set of interconnected hypotheses. We hypothesize that the long-term collaboration inherent in PjBL acts as a social engine that forces learners to resolve conflicts and share cognitive loads, thereby significantly predicting gains in Collaborative Competence (Ningsih et al., 2025). Concurrently, we posit that the requirement for presenting tangible products serves as a specialized catalyst for Mathematical Communication Competence. Because students must defend their findings using the "mathematics register," the PjBL framework effectively transforms abstract reasoning into a discursive activity, fostering a deeper mastery of mathematical language than traditional lecture-based methods (Rocha & Babo, 2024; Chen & Yang, 2019). Ultimately, the success of these outcomes is hypothesized to be moderated by the quality of institutional support and meticulous project planning, which mitigate the common structural barriers of workload and resource scarcity (Hussein, 2021; Wijnia et al., 2024)

METHOD

Conceptual Model and Hypothesis Development

Project design Characteristics (PDC)

Project design features play a key role in determining the quality of students' learning experiences and the development of core competencies in the context of Project-Based Learning (PjBL). A well-designed project provides learners with a clear goal, an authentic problem, and a meaningful task, thereby stimulating inquiry and collaboration (Markula & Aksela, 2022; Ferrero et al., 2021). A clear project construct-with clearly defined stages such as planning, investigating, implementing, and presenting-helps guide the learning process and increase students' ownership of the learning process. Therefore, a well-designed project in mathematics teaching has the potential to promote both collaborative and expressive competencies, helping learners connect conceptual understanding with communication and coordination skills. Based on the above theoretical foundations, the following research hypotheses are proposed:

Hypothesis 1 (H1): Project Design Characteristics (PDC) have a significant positive impact on students' Collaborative Competence (COC).

Hypothesis 2 (H2): Project Design Characteristics (PDC) significantly and positively influence students' Mathematical Communication Competence (MCC).

Teachers' support

In the PjBL framework, the teacher's role transcends traditional instruction, shifting toward **facilitation and adaptive scaffolding**. According to Zhong and Lyu (2022),

providing timely feedback, provocative questioning, and structural guidance is essential for enhancing student motivation and autonomy. This support acts as a catalyst for a collaborative environment rooted in mutual respect, enabling students to navigate group conflicts and model precise mathematical reasoning through reflective discussion. Ultimately, consistent and well-structured teacher intervention serves as the "bridge" that allows students to transition from passive learners to active collaborators who can confidently share and articulate complex mathematical concepts. Based on this, we formulated the following hypotheses:

Hypothesis 3 (H3): Teachers' support (TSP) has a significant positive impact on students' collaborative competence (COC).

Hypothesis 4 (H4): Teachers' support (TSP) has a significant positive impact on students' Mathematical Communication Competence (MCC).

Collaborative Environment

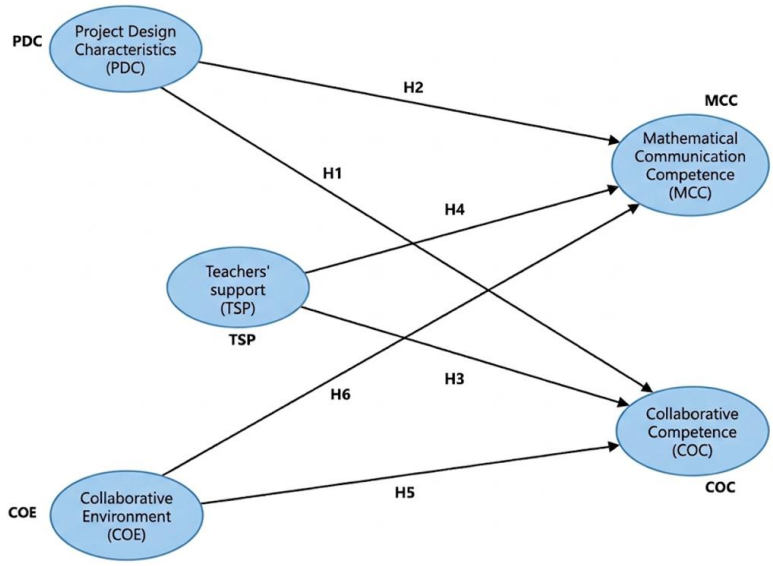
In PjBL, a robust collaborative environment is fundamental to fostering student engagement and the co-creation of knowledge. This environment—characterized by psychological safety, inclusivity, and shared goals—enables students to resolve conflicts and maintain continuous commitment to teamwork (Hussein, 2021). Within such a context, collaboration transcends being a mere tool for project completion; it becomes a transformative process of mutual support and peer learning. Furthermore, a well-structured collaborative environment in mathematics education empowers students to take ownership of their learning, significantly boosting both academic achievement and motivation (Holmes & Hwang, 2016). By explaining reasoning, defending methodologies, and building upon the ideas of others, students naturally bridge the gap between collaborative competence and mathematical communication competence. Ultimately, an encouraging environment provides the essential foundation for students to confidently articulate mathematical arguments and organize complex group tasks with clarity and coherence. Based on this, we formulated the following hypotheses:

Hypothesis 5 (H5): Collaborative Environment (COE) has a significant positive impact on students' Collaborative Competence (COC).

Hypothesis 6 (H6): Collaborative Environment (COE) has a significant positive impact on students' Mathematical Communication Competence (MCC).

Figure 1 depicts the proposed conceptual model based on the hypotheses. Each factor is represented by an ellipse, and the prediction is denoted by an arrow.

Figure 1
Research model



Research Design

This study employs an explanatory quantitative research design to systematically examine the causal relationships between pedagogical factors and the development of core competencies within the PjBL framework. Data were collected from high school and mathematics education students in Vietnam and analyzed using Partial Least Squares Structural Equation Modeling (PLS-SEM) via SmartPLS software. The selection of PLS-SEM is justified by its robust predictive capabilities and its effectiveness in theory-building, particularly when handling medium sample sizes common in educational research. The proposed structural model evaluates three antecedent factors—Project Design Characteristics (PDC), Teachers' Support (TSP), and Collaborative Environment (COE)—as primary predictors hypothesized to directly influence Collaborative Competence (COC) and Mathematical Communication Competence (MCC).

Participants and data gathering tools

This study utilized a total sample of 1,089 valid participants, a size strategically determined to ensure robust statistical power and high predictive accuracy for the complex structural model. To achieve a representative sampling frame, the study targeted five high schools and four pedagogical universities across Hanoi, Thanh Hoa, and Da Nang in Vietnam.

A stratified random sampling procedure was implemented: after stratifying institutions by public/private status and grade level, specific classes and students were randomly

selected from each stratum. The final sample comprises two distinct cohorts: 835 high school students (grades 10–12) and 254 mathematics education majors (sophomore to senior levels). This dual-cohort approach provides a comprehensive perspective from both current learners and future educators.

Data were gathered through a structured online survey via Google Forms to ensure standardized collection. A total of 1,231 surveys were distributed, and 1,103 were returned, yielding 1,089 valid responses (a response rate of 98,73%). This large-scale observation satisfies the requirements for PLS-SEM analysis, providing the necessary stability for structural paths and complex model estimation without compromising statistical significance. The instrument utilized Likert-scale items adapted from validated research to capture perceptions of Project Design Characteristics (PDC), Teachers' Support (TSP), and Collaborative Environment (COE).

Research Instrument

The primary data were collected using a structured research instrument comprising five key latent constructs measured through 28 observed indicators, adapted from validated scales in the field of Project-Based Learning (PjBL). The measurement model facilitates a comprehensive analysis of both exogenous and endogenous variables. First, Project Design Characteristics (PDC), measured by six indicators (PDC1–PDC6), assesses the extent to which educators align instructional materials and learning objectives with students' diverse readiness and interests. Second, Teachers' Support (TSP), comprising seven indicators (TSP1–TSP7), evaluates the pedagogical guidance and flexibility provided to ensure students effectively internalize core concepts. Third, Collaborative Environment (COE) is defined by four indicators (COE1–COE4) to measure the degree to which teachers facilitate multifaceted interaction and flexible learning spaces. Finally, the model evaluates two primary endogenous constructs: Collaborative Competence (COC) and Mathematical Communication Competence (MCC). These variables quantify the impact of PjBL strategies on students' ability to work effectively in teams and their proficiency in expressing mathematical ideas, providing a comprehensive assessment of the model's pedagogical effectiveness.

The descriptors below are designed to measure aspects of the construct, content, and challenge of the Mathematics project based on student perceptions or teacher assessment.

Table 1
Constructs and Items

Construct	Total Items	Item	Scale items
Project Design Characteristic (PDC)	6	PDC1	Project objectives and output (product) requirements are clearly defined at the outset.
		PDC2	The project poses moderate challenges and encourages students to think more deeply about their mathematical knowledge.
		PDC3	The project requires students to apply mathematical knowledge to solve a real-world or meaningful problem.
		PDC4	The project requires students to integrate and use knowledge from many different topics/subjects (interdisciplinary).
		PDC5	Projects allow students some choice and autonomy in how they work or the final product.
		PDC6	The project has a clear phased construct (e.g., research, planning, implementation, presentation).
Teacher Support (TSP)	7	TSP1	Teachers provide timely and specific feedback on group progress and products.
		TSP2	The teacher gives directional suggestions instead of giving a direct answer when the group is stuck.
		TSP3	Teachers create opportunities for students to explore, discover and solve problems on their own.
		TSP4	Teachers provide constructive feedback on group work products and processes.
		TSP5	Teachers facilitate groups to search for and use various resources and support tools.
		TSP6	Teachers help students connect the mathematical knowledge in the project to real-world applications or other subjects.
		TSP7	Teachers closely monitor the work of groups and intervene when there are conflicts or members are left out.
Collaborative Environment (COE)	4	COE1	Team members feel comfortable and safe sharing ideas, even incomplete ones.
		COE2	Members trust in the abilities and responsibilities of others in the group.
		COE3	The team has a clear working process (e.g., how to make decisions, how to resolve disagreements).
		COE4	The team maintains positive and friendly relationships, which helps work more effectively.
Collaborative Competency (COC)	6	COC1	Students learn how to resolve conflicts and maintain group unity.
		COC2	Students are always proactive in coming up with ideas and suggestions to solve project problems.
		COC3	Students complete assigned work in groups on time and with quality.
		COC4	Students know how to discuss and negotiate to reach a common consensus of the whole group.
		COC5	Students actively encourage other members to participate and contribute their opinions.
		COC6	Students are willing to share their learning materials, resources and experiences with other members.
Communication competency (MCC)	5	MCC1	Students are able to defend and debate their mathematical arguments.
		MCC2	Students can clearly explain mathematical concepts and rules used in projects to others.
		MCC3	Students are able to accurately use mathematical symbols, terminology, and language in project reports and presentations.
		MCC4	Students can express their ideas and solutions in many different forms (speech, diagrams, tables, formulas).
		MCC5	Students are able to analyze, evaluate and critique mathematical arguments of their peers in groups or when presenting.

FINDINGS

Measurement Model Assessment

Convergent validity and reliability

As summarized in Table 2, the measurement model demonstrates exceptional internal consistency and convergent validity. The Cronbach's alpha and Composite Reliability (rho_a and rho_c) for both MCC and COC significantly exceed the 0.70 and 0.95 benchmarks, respectively, indicating high scale reliability. Furthermore, the Average Variance Extracted (AVE) values (0.842 for MCC; 0.834 for COC) surpass the 0.50 threshold, confirming that the latent constructs account for over 80% of the variance in their indicators. These robust psychometric properties provide a rigorous foundation for the subsequent structural equation modeling (SEM) analysis.

Table 2
Construct reliability and validity

	Cronbach's alpha	Composite reliability (rho_a)	Composite reliability (rho_c)	Average variance extracted …
MCC	0.953	0.953	0.964	0.842
COC	0.960	0.960	0.968	0.834

Discriminant validity

The discriminant validity of the measurement model was strictly confirmed using the Fornell-Larcker criterion (Table 3). The results show that the square root of the Average Variance Extracted (AVE) for both COC (0.913) and MCC (0.917) are consistently higher than the correlation coefficient between these two constructs (0.857). This empirical evidence demonstrates that each latent variable shares more variance with its own indicators than with any other construct in the model, thereby establishing strong discriminant validity for the study.

Table 3
Fornell-Larcker

Item	COC	MCC
COC	0.913	
MCC	0.857	0.917

The discriminant validity of the measurement model was further validated through an examination of cross-loadings (Table 4). The results indicate that the outer loading of each indicator on its designated latent construct is consistently and significantly higher than its cross-loadings on any other constructs. Specifically, the indicators for COC (0.907–0.917), COE (0.905–0.936), MCC (0.908–0.925), PDC (0.881–0.911), and TSP (0.893–0.927) all show the strongest correlations with their respective variables. This consistent pattern confirms that each construct is empirically unique and that the measurement items represent distinct conceptual dimensions, fulfilling the statistical requirements for a robust structural model.

Table 4
Cross Loadings

	COC	COE	MCC	PDC	TSP
COC1	0.911	0.810	0.786	0.768	0.791
COC2	0.916	0.802	0.779	0.757	0.790
COC3	0.907	0.806	0.780	0.777	0.808
COC4	0.916	0.803	0.775	0.774	0.809
COC5	0.917	0.795	0.785	0.762	0.796
COC6	0.912	0.797	0.789	0.756	0.804
COE1	0.795	0.905	0.727	0.770	0.806
COE2	0.796	0.914	0.742	0.768	0.804
COE3	0.826	0.936	0.747	0.797	0.834
COE4	0.819	0.930	0.746	0.804	0.837
MCC1	0.769	0.717	0.912	0.709	0.720
MCC2	0.781	0.735	0.925	0.715	0.744
MCC3	0.798	0.743	0.919	0.731	0.752
MCC4	0.790	0.747	0.922	0.726	0.750
MCC5	0.789	0.742	0.908	0.742	0.748
PDC1	0.761	0.777	0.722	0.911	0.797
PDC2	0.739	0.775	0.724	0.898	0.794
PDC3	0.744	0.751	0.690	0.881	0.785
PDC4	0.753	0.753	0.705	0.895	0.771
PDC5	0.755	0.765	0.718	0.905	0.783
PDC6	0.771	0.788	0.710	0.910	0.809
TSP1	0.790	0.809	0.715	0.793	0.893
TSP2	0.785	0.798	0.721	0.791	0.894
TSP3	0.810	0.826	0.752	0.817	0.927
TSP4	0.801	0.808	0.730	0.794	0.908
TSP5	0.799	0.810	0.735	0.796	0.910
TSP6	0.787	0.806	0.740	0.791	0.906
TSP7	0.790	0.803	0.740	0.782	0.908

Table 5
Heterotrait-monotrait ratio (HTMT)

	Heterotrait-monotrait ratio (HTMT) - List
COC < - > MCC	0.895

The HTMT value of 0.895 reflects the average correlation between the observed variables of the two constructs (see Table 5). This value falls between the strict (0.85) and practical (0.90) acceptance thresholds set by Henseler et al. (2015). This result indicates a strong, empirically reasonable relationship between COC and MCC within the PjBL context, but does not exceed the allowable level for conceptual overlap. The close connection is theoretically justified, as group work necessitates both coordination (COC) and clear expression/exchange of ideas (MCC).

Collinearity analysis

The collinearity assessment of the outer model was conducted by evaluating the Variance Inflation Factor (VIF) for all indicators. As shown in Table 6, the VIF values for all items across the five constructs (COC, COE, MCC, PDC, and TSP) range from 3.344 to 4.563. These values are consistently below the conservative threshold of 5.0, indicating that multicollinearity among the indicators does not pose a threat to the

measurement model. Consequently, all observed indicators are statistically suitable for further structural analysis without redundant overlap.

Table 6
Collinearity statistics (VIF)-Outer model–List

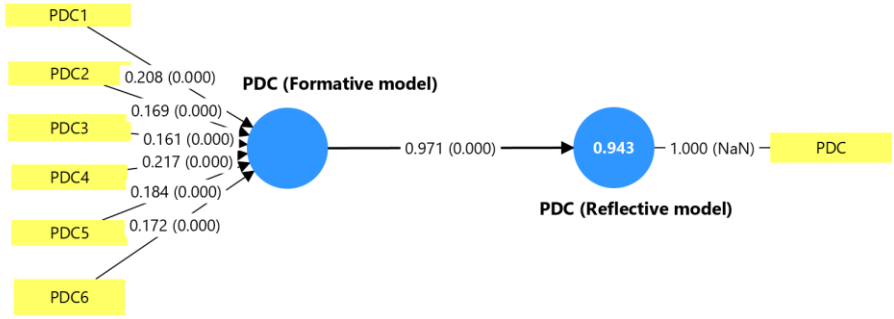
Item	VIF	Item	VIF
COC1	4.256	MCC5	3.693
COC2	4.398	PDC1	4.178
COC3	4.028	PDC2	4.124
COC4	4.405	PDC3	3.629
COC5	4.419	PDC4	3.344
COC6	4.345	PDC5	3.836
COE1	3.535	PDC6	3.803
COE2	3.821	TSP1	4.135
COE3	4.431	TSP2	4.018
COE4	3.837	TSP3	4.563
MCC1	3.940	TSP4	4.455
MCC2	4.527	TSP5	4.409
MCC3	4.203	TSP6	4.245
MCC4	4.371	TSP7	4.114

Table 7
Collinearity statistics (VIF) - PDC

Item	VIF
PDC	1.000
PDC1	4.178
PDC2	4.124
PDC3	3.629
PDC4	3.344
PDC5	3.836
PDC6	3.803

PDC3 (VIF = 3.629) and PDC4 (VIF = 3.344) have lower VIF values, indicating lower collinearity and more independent information contribution within the structure (see Table 7). Generally, VIF fluctuations in the range of 3.3–4.2 show a moderate relationship, ensuring internal consistency without data duplication. This confirms that the observed variables in the PDC structure are closely correlated but not redundant, effectively reflecting different project design aspects. The average VIF level of about 3.8 indicates good explanatory ability for the latent concept without negatively affecting the PLS-SEM model estimation. Thus, the PDC structure has no multicollinearity problems, ensuring measurement model reliability and stable outer weights, and confirming that the construct's indicators effectively and independently reflect core PjBL project design competence.

Figure 2
Validity Assessment of PDC Higher-Order Model



The validity of the Higher-Order Model (PDC) is empirically supported by both formative and reflective assessments. All six formative indicators (PDC1–PDC6) exhibit positive and statistically significant outer weights ($p < 0.001$), confirming their distinct and essential contributions to the construct. Furthermore, the strong path coefficient (0.971) and high outer loading (0.943) between the formative composite and the reflective latent variable demonstrate excellent explanatory power and redundancy validity. These results confirm that the PDC model is robust, reliable, and statistically sound for further structural analysis.

Collinearity assessment for the Teacher Support (TSP) construct (Table 8) shows VIF values for all indicators (TSP1–TSP7) ranging from 4.018 to 4.563. With an average of 4.30 and all individual values remaining below the 5.0 threshold, there is no evidence of critical multicollinearity. These results confirm a healthy level of internal correlation among the indicators, ensuring the informational independence and structural integrity of the TSP construct within the PjBL framework.

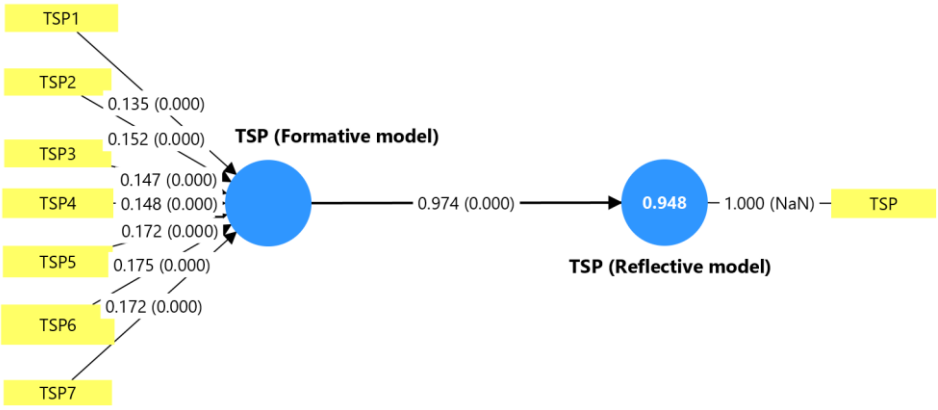
Table 8
Collinearity statistics (VIF) - HTGV

	VIF
TSP	1.000
TSP1	4.135
TSP2	4.018
TSP3	4.563
TSP4	4.455
TSP5	4.409
TSP6	4.425
TSP7	4.114

The redundancy validity of the Higher-Order Model for Teacher Support (TSP) is empirically confirmed (see Figure 3). All seven formative indicators (TSP1–TSP7) exhibit positive and highly significant outer weights (ranging from 0.135 to 0.175; $p < 0.001$), validating their specific contributions to the construct. The strong path coefficient of 0.974 ($p = 0.000$) between the formative and reflective components, coupled with an R^2 value of 0.948, demonstrates exceptional explanatory power and

convergence. These results indicate that the higher-order TSP construct is robustly defined and statistically sound for structural analysis.

Figure 3
Validity Assessment of TSP Higher-Order Model

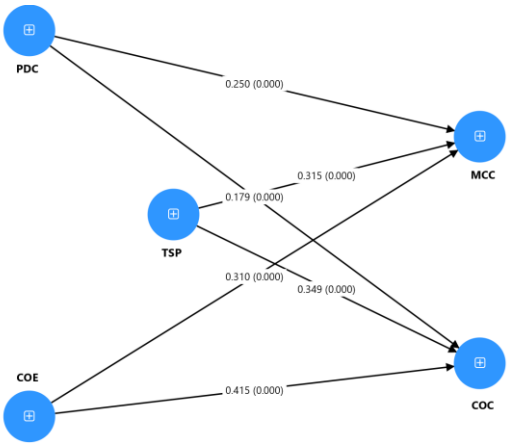


Structural Model Assessment

The effects in the model

The structural model analysis confirms that all hypothesized paths are positive and statistically significant ($p < 0.001$), demonstrating the robust predictive power of the PjBL framework (see Figure 4). COE emerges as the strongest predictor for both endogenous constructs, exerting a substantial impact on COC) ($\beta = 0.415$) and Mathematical Communication Competence (MCC) ($\beta = 0.310$). Furthermore, TSP and PDC significantly enhance student outcomes, with TSP showing a particularly strong influence on COC ($\beta = 0.349$). These results suggest that while project design and pedagogical guidance are essential, the cultivation of a synergistic collaborative space is the primary driver for developing advanced mathematical and interpersonal competencies in project-based learning.

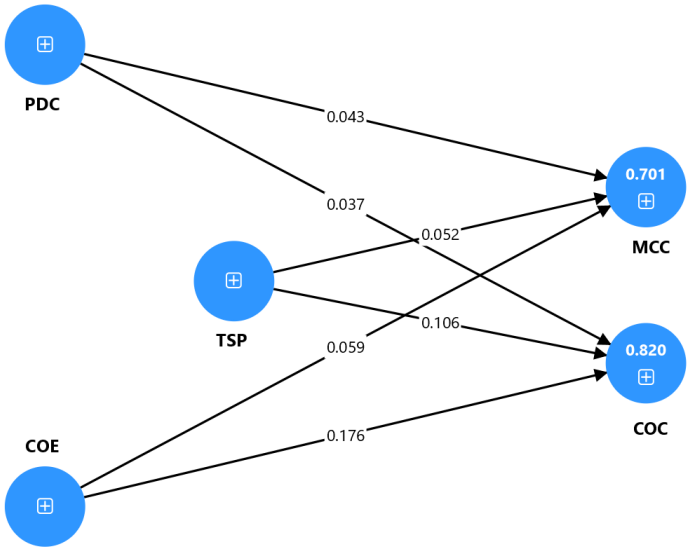
Figure 4
The effects in the model (Source: Authors' own elaboration)



The effects sizes

As illustrated in Figure 5, COC has R^2 is 0.820 (very high), demonstrating the model's suitability and the factors' strong predictive ability for cooperation capacity development. Appropriate teacher design, active student support, and an open environment increase interaction, sharing, and problem-solving, thereby developing cooperation capacity. MCC is also strongly influenced by project-based teaching (R^2 = 0.701). By promoting regular idea presentation, group discussion, and verbal/product-based explanation of concepts, communication capacity is strengthened. TSP has f^2 = 0.176 (affirming the central role of teachers in PjBL implementation). Teachers instruct, support motivation, and coordinate groups. PDC (f^2= 0.106) and COE (f^2 = 0.052) show a small to medium, yet fundamental, impact, creating conditions for effective PjBL. The model is convincing and reflects PjBL practice well. PjBL success in developing collaborative and mathematical communication competence relies significantly on the teacher's active supporting role. Promoting self-assessment, feedback, and open idea exchange strongly develops students' mathematical communication and collaboration skills.

Figure 5
Results of the SEM Analysis: R-Square and Effect Size (f^2)



Model Fit Assessment

The structural model assessment (Table 9) confirms a robust alignment between the hypothesized framework and empirical classroom data. The low SRMR (0.027) and discrepancies (d_ULS = 0.302; d_G = 0.231) demonstrate that the model accurately captures the nuanced interactions of project-based learning. While the Chi-square reflects the sensitivity of the large educational sample, the high NFI (0.966) validates that the proposed paths are highly effective in explaining student development. Scientifically, these results prove that the integration of PDC, TSP, and COE is not merely a theoretical construct but a statistically validated driver of student outcomes. This strong model fit provides a reliable foundation for educators to implement these strategies, ensuring that pedagogical interventions effectively foster mathematical communication and collaborative competencies

Table 9
Model Fit

	Saturated model	Estimated model
SRMR	0.015	0.027
d_ULS	0.096	0.302
d_G	0.185	0.231
Chi-square	1111.085	1317.083
NFI	0.972	0.966

Analyzing the predictive relevance power of regression model

Determining the predictive relevance power of regression model

Table 10
PLSpredict LV summary - PLS-SEM

Item	Q^2 predict	RMSE	MAE
MCC	0.696	0.553	0.387
COC	0.816	0.431	0.278

The predictive relevance of the structural model was evaluated using the PLSpredict procedure (Table 10). The results show that all $Q^2_{predict}$ values for the key endogenous constructs-MCC (0.696) and COC (0.816)-are substantially positive, confirming that the model has high out-of-sample predictive power. Scientifically, this validates that the relationship between PjBL design and student competencies is consistently reproducible beyond the current dataset. In practice, the low error rates (RMSE and MAE) suggest that the model can accurately forecast the development of students' mathematical communication and collaborative skills. For educators, these findings provide a reliable tool to predict the effectiveness of pedagogical interventions, ensuring that project-based strategies lead to measurable and sustainable improvements in student learning outcomes.

PLSpredict MV summary-Overview

Table 11 evaluates the model's predictive precision at the indicator level for both Mathematical Communication (MCC) and Collaboration (COC) competencies. The $Q^2_{predict}$ values for all indicators range from 0.553 to 0.691, significantly exceeding the 0.35 threshold and confirming strong predictive relevance. Notably, the COC group (0.670–0.691) shows the highest predictive accuracy, particularly in forecasting students' coordination and mutual support within group activities. The MCC group (0.553–0.599) also demonstrates robust performance, accurately predicting how students express and communicate mathematical ideas in PjBL contexts. These results prove that the model is not only theoretically sound but also a highly reliable tool for forecasting specific manifestations of student competency growth.

Table 11
PLSpredict MV summary – Overview

	Q^2predict	PLS-SEM_RMSE	PLS SEM_MAE	LM_RMSE	LM_MAE	IA_RMSE	IA_MAE
COC1	0.681	0.511	0.334	0.523	0.337	0.905	0.624
COC2	0.670	0.510	0.327	0.519	0.334	0.888	0.600
COC3	0.691	0.497	0.321	0.503	0.322	0.894	0.582
COC4	0.689	0.488	0.330	0.495	0.333	0.875	0.568
COC5	0.670	0.519	0.327	0.521	0.328	0.904	0.585
COC6	0.675	0.518	0.336	0.522	0.334	0.909	0.596
MCC1	0.553	0.599	0.423	0.605	0.430	0.896	0.665
MCC2	0.579	0.592	0.416	0.598	0.423	0.912	0.677
MCC3	0.597	0.576	0.390	0.581	0.391	0.906	0.637
MCC4	0.596	0.562	0.392	0.570	0.396	0.884	0.627
MCC5	0.599	0.564	0.421	0.569	0.421	0.890	0.677

Analyzing cross-validated predictive ability of the model PLS-SEM vs. Indicator average (IA)

The predictive validity of the structural model was rigorously assessed using the Cross-Validated Predictive Ability Test (CVPAT). As shown in Table 12, the PLS-SEM model consistently exhibits significantly lower loss values compared to the Indicator Average (IA) benchmark across all constructs (Average Loss Difference from -0.471 to -0.545; $p < 0.001$). Furthermore, Table 13 reveals that PLS-SEM also outperforms the Linear Model (LM) with statistically significant differences for both MCC ($p = 0.003$) and COC ($p < 0.001$). Collectively, these results demonstrate that the proposed model possesses superior out-of-sample predictive power and higher explanatory efficiency than both naive and linear benchmarks, confirming its robustness for diagnosing and forecasting student competency development in PjBL contexts.

Table 12
CVPAT LV summary - PLS-SEM vs. Indicator average (IA)

	PLS loss	IA loss	Average loss difference	t-value	p-value
MCC	0.335	0.806	-0.471	10.611	0.000
COC	0.257	0.803	-0.545	11.539	0.000
Overall	0.293	0.804	-0.512	11.383	0.000

Analyzing cross-validated predictive ability of the model PLS-SEM vs. Linear Model (LM)

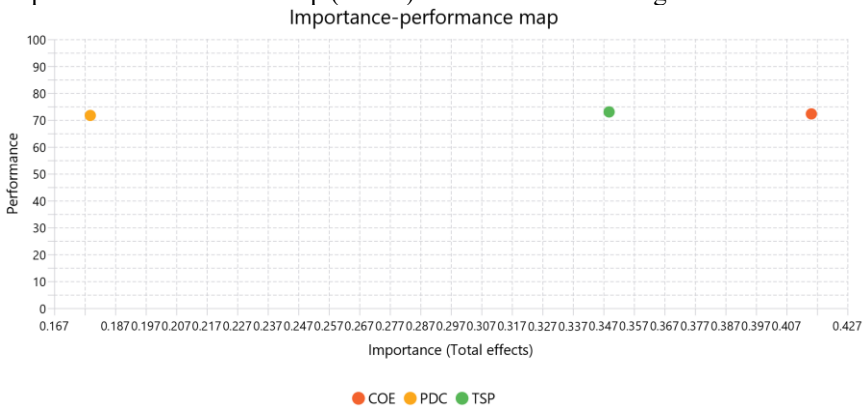
Table 13
CVPAT LV summary - PLS-SEM vs. Linear Model (LM)

	PLS loss	LM loss	Average loss difference	t-value	p-value
MCC	0.335	0.342	-0.007	2.930	0.003
COC	0.257	0.264	-0.007	3.603	0.000
Overall	0.293	0.300	-0.007	4.999	0.000

Importance-Performance Map Analysis (IPMA)

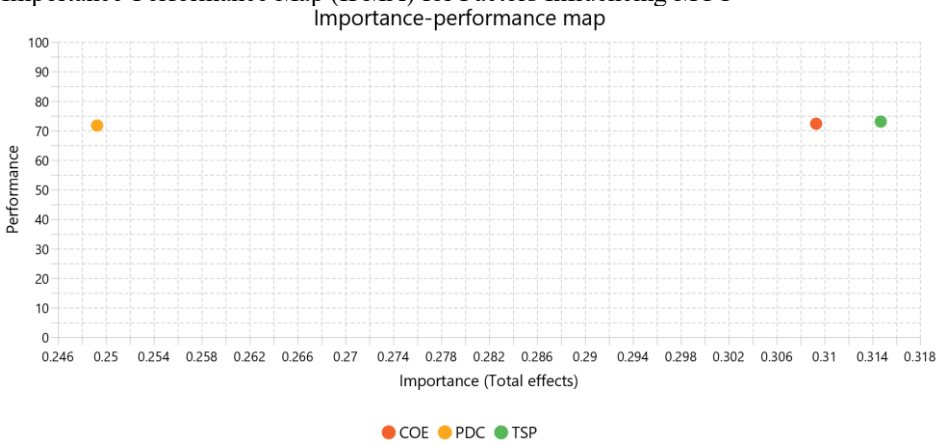
Importance-Performance Map Analysis (IPMA)

Figure 6
Importance-Performance Map (IPMA) for Factors Influencing COC



The Importance-Performance Map Analysis (IPMA) provides a strategic priority matrix for enhancing COC. As illustrated in Figure 6, Collaborative Environment (COE) stands out as the most critical factor, exhibiting the highest total effect (Importance ≈ 0.415) while maintaining a high performance score (≈72%). Teacher Support (TSP) also shows substantial importance (≈ 0.347) with the highest performance level (≈74%), indicating its established effectiveness in the current PjBL framework. In contrast, Project Design Characteristics (PDC) demonstrates the lowest importance (≈0.170) despite having a performance score comparable to COE. Scientifically, these results prioritize the optimization of the collaborative atmosphere and pedagogical support over structural project design to maximize the development of and collaboration.

Figure 7
Importance-Performance Map (IPMA) for Factors Influencing MCC



The Importance-Performance Map Analysis (IPMA) identifies Teachers' Support (TSP) as the primary structural driver for Communication Competency (MCC) (see Figure 7), exhibiting peak values in both total effects (≈ 0.315) and execution (≈73%). Collaborative Environment (COE) follows as a high-leverage antecedent (Importance ≈ 0.309; Performance ≈ 72%), confirming that interpersonal dynamics are more statistically significant than structural factors. Conversely, Project Design Characteristics (PDC), despite comparable performance, shows the lowest relative importance (Importance ≈ 0.249), indicating a diminishing marginal return on purely architectural project adjustments.

Importance-Performance Map at the Construct Level

Table 15
Construct total effects and Construct performances for MCC and COC

	Total effects	Performance	Total effects	Performance
	MCC		COC	
COE	0.310	72.258	0.415	72.258
PDC	0.250	71.674	0.179	71.674
TSP	0.315	72.985	0.349	72.985

The IPMA results (Table 15) reveal distinct structural prioritizations for Communication Competency (MCC) and Collaborative Competency (COC). For MCC, Teachers' Support (TSP) serves as the primary driver (Total Effects = 0.315; Performance = 72.985%), closely followed by Collaborative Environment (COE). Conversely, COE exerts a significantly higher influence on COC (Total Effects = 0.415), indicating that the social atmosphere is more critical for collaborative output than for communication alone. In both models, Project Design Characteristics (PDC) demonstrates the lowest relative importance (0.250 for MCC; 0.179 for COC) despite consistent performance levels (71.7%). Strategically, these findings prioritize pedagogical guidance (TSP) to maximize MCC and environmental optimization (COE) to bolster COC, as socio-pedagogical elements consistently outweigh structural project design in fostering latent student competencies.

Implications for Sustainable Development Goals (SDGs)

This study provides robust empirical evidence linking PjBL to the United Nations SDGs 4 and 11. By bridging theory and real-world application, PjBL effectively embeds sustainability into disciplinary training (Navarro et al., 2025; Zhang et al., 2024). In the Vietnamese context, the statistical significance of PjBL implementation (PDC) and teacher support (TSP) confirms that active engagement—rather than passive acquisition—enables students to leverage mathematical reasoning for social well-being. The substantial effect sizes of COC and MCC identified through the SEM model directly address 21st-century demands, transforming learners into active participants ready for digital-age challenges (Zhang et al., 2023; Ningsih et al., 2025). Furthermore, by focusing on contextually relevant STEM problems, PjBL promotes educational equity and inclusivity, offering a platform for diverse learners to excel (Craig et al., 2019; Rehman et al., 2024). Ultimately, PjBL serves as a transformative tool that fosters learner autonomy and prepares a new generation of innovators for a sustainable future (Jeong, 2025).

DISCUSSION

The empirical findings of this study provide robust support for the theoretical tenets of constructivism and experiential learning, reinforcing the principle that mathematical competencies are most effectively cultivated within interactive, context-rich, and reflective pedagogical frameworks (Huang, 2021; Kolb, 2014). Analysis of the structural model reveals a clear hierarchy of influence: Collaborative Environment (COE) stands as the primary driver for both Collaborative Competence (COC, β = 0.407) and Mathematical Communication Competence (MCC, β = 0.385), followed by Teachers' Support (TSP, β = 0.291) and Project Design Characteristics (PDC, β = 0.201). To capture a more holistic view of these dynamics, the study extends beyond direct path coefficients by incorporating Importance-Performance Map Analysis (IPMA). Unlike standard β coefficients which isolate direct links, the 'importance' metric in IPMA accounts for the total effects—integrating both direct and indirect structural pathways—while simultaneously factoring in the actual performance of each construct. This systemic approach offers a superior diagnostic value for educators, as it identifies which pedagogical interventions yield the highest cumulative impact on

student outcomes. The substantial explanatory power of the model (R^2 for COC = 0.820; R^2 for MCC = 0.701) underscores that the synergy between a well-structured collaborative milieu and proactive teacher guidance is the decisive mechanism for translating PjBL initiatives into tangible 21st-century skills.

A central finding of this study is that teacher support (TSP) emerged as the strongest predictor of both communication and collaborative competencies, surpassing project design and environment. This underscores the teacher's pivotal role as a "scaffolder" (Wang et al., 2024), particularly in the Vietnamese cultural context where students often require instructor validation to overcome psychological barriers and engage in independent mathematical reasoning (Nga, 2026; Makamure, 2025). Furthermore, the high correlation between COC and MCC suggests that mathematical communication is not merely a byproduct of collaboration but its primary vehicle for constructing shared cognitive spaces (Almulla, 2020). However, the substantial effect sizes also raise critical questions regarding discriminant validity in self-reported data; future research should employ granular behavioral observations to more clearly differentiate between the act of "working together" and the specific cognitive process of "mathematical communication".

The significant influence of project design underscores that the "authenticity" of the task is a catalyst for engagement. Projects that are well-structured with interdependent tasks-where one student's output is necessary for another's success-naturally foster a need for clear mathematical communication (Zhang & Ma., 2023). This is complemented by the collaborative environment, which acts as a social motivator by creating a space where misunderstandings are clarified through peer-to-peer dialogue.

Despite these promising results, several limitations must be considered. The cross-sectional design of this study provides only a snapshot in time, whereas MCC and COC are developmental skills that may evolve differently over multiple project cycles. Additionally, the reliance on self-reported measures may introduce social desirability bias, where students overrate their competencies to align with perceived classroom expectations. Future research should incorporate objective assessments, such as rubric-based expert evaluations or longitudinal tracking, to triangulate these findings. In conclusion, while PjBL is a powerful pedagogical tool for developing 21st-century skills, its success depends heavily on the strategic alignment of teacher guidance with the socio-cultural dynamics of the Vietnamese classroom.

CONCLUSION

This study confirms that Project-Based Learning (PjBL) is a robust pedagogical framework for enhancing mathematical communication (MCC) and collaborative competence (COC) within the Vietnamese educational context. The PLS-SEM analysis underscores that the synergy between intentional project design, a supportive environment, and proactive teacher guidance is essential for fostering these core competencies. Practical implications suggest that educators should adopt a "fading guidance" strategy, transitioning from knowledge transmitters to scaffolders who gradually foster learner autonomy. Curriculum designers should emphasize "positive interdependence" to ensure collective problem-solving, while administrators should

prioritize professional development focused on dialogic facilitation. Despite the strong predictive power of the large-scale sample (N = 1,089), this study is limited by its cross-sectional design and reliance on self-reported data. Future research should employ longitudinal designs or mixed-method approaches, such as video-based discourse analysis, to objectively capture the developmental mechanics of student interaction. Ultimately, the success of PjBL depends on the strategic alignment of pedagogical design with the evolving needs of the learners.

DECLARATIONS

Conflict of Interests

The authors declare no conflict of interest.

Funding

This research received no external funding.

Ethical statement

This study was deemed exempt from the requirement for formal approval from an Ethics Committee.

Data Availability

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

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