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Integrating Critical Thinking and Mathematical Inquiry: A Framework for Instruction in Ordinary Differential Equations

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Critical thinking is widely recognised as a core educational objective in higher educa-tion. This study examined how undergraduate students' critical thinking can be en-hanced through mathematics by proposing a teaching framework that integrates critical thinking with mathematical inquiry in the context of ordinary differential equations (ODEs). A quasi-experimental mixed-methods design was employed with 98 students from one university, combining quantitative and qualitative data to investigate the ef-fects of different instructional approaches on students' critical thinking development and ODE learning.The findings showed that inquiry-based instruction supported students' conceptual understanding of ODEs and significantly improved their critical thinking skills. However, instructional approaches that explicitly infused critical thinking into inquiry-based instruction were more effective than implicit inquiry approaches in im-proving students' overall critical thinking and ODE achievement. In addition, students' critical thinking performance was strongly positively correlated with their ODE achievement. Qualitative findings further suggested that explicit critical thinking in-struction may help students transfer critical thinking to real-world problem solving and foster their critical thinking disposition.

Keywords: critical thinking; inquiry-based instruction; 5E model; Ordinary Differential Equations; engineering students

INTRODUCTION

Critical thinking (CT) is widely recognized as an essential ability for 21st-century learners in contemporary society. Scriven and Paul (1987) noted that CT requires applying, analyzing, synthesizing, and/or evaluating information gathered from or generated by observation, experience, reflection, reasoning, or communication as a

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guide to belief and action. It is a vital tool for distinguishing fact from opinion, enabling thorough analysis, evaluation, and integration in strategic decision-making (Bonet et al., 2022; Ennis, 1989; Facione, 2000; Paul & Elder, 2019b; Ricci, 2014). In recent decades, researchers have explored effective methods for cultivating CT among university students.

CT is an intellectually disciplined process that includes the active employment of skills for comprehension, application, analysis, synthesis, observation, interpretation of experience, reflection, reasoning, and communication, all of which are emphasized in mathematics learning(Paul & Elder, 2019a). Glazer (2001) argued that CT in mathematics refers to the ability to generalize, prove, or evaluate less well-known mathematical problems requiring prior knowledge, mathematical reasoning, and cognitive processes. Dolapcioglu and Doğanay (2022) claimed that CT in mathematics entails not only knowing and using knowledge to reach correct solutions but also understanding, interpreting, investigating different ways of finding solutions, and reflecting on the benefits of mathematics in everyday life.Several scholars believe that someone talented in mathematics will be good at thinking, and someone trained in mathematics will become a good thinker(Cresswell & Speelman, 2020; Li & Schoenfeld, 2019). Mathematics instruction is often primarily concerned with students' conceptual understanding and their ability to apply that understanding to solve problems accurately. An essential component that bridges these two objectives is a deep understanding of mathematics, which necessarily requires instructors to actively probe students' CT processes. Cognitively Guided Instruction (CGI) theory, developed by Carpenter et al. (1999), holds that listening to and acting on students' thinking enables instructors to tailor instruction to support the development of deep conceptual understanding (Carpenter, Fennema, Franke, Levi, & Empson, 1999). This view aligns with Dolapcioglu & Doğanay (2022), who claim that cultivating CT in mathematics involves not only knowing and using knowledge to reach correct solutions, but also understanding, interpreting, and investigating different ways of finding solutions, and reflecting on the application of mathematics in everyday life. Evendi et al. (2022) presented that CT skills are needed as cognitive bridging to understand and solve problems in mathematics; there is a significant and interrelated relationship between CT and learners' academic achievement.

CT does not automatically emerge from a mathematics curriculum; instead， it is fostered through teaching approaches that encourage students' reasoning and questioning while they are engaged in intellectually challenging tasks (Jablonka, 2014). According to Andreucci-Annunziata et al. (2023), CT can be trained through targeted teaching strategies. Several scholars have argued that instructional practices that encourage inquiry, dialogue, and reflection foster a learning environment that promotes deeper cognitive engagement in mathematics (Kuster, Johnson, Keene, & Andrews-Larson, 2018; Laursen & Rasmussen, 2019). For example, Doz et al.(2024) reported that problem-based mathematics instruction encourages students to apply prior knowledge to more complex problems and is intended to develop critical thinking and problem-solving skills. In addition, Muhibbuddin et al. (2023) found that instruction

emphasizing higher-order questions significantly improved students' critical thinking compared with conventional teaching.

Inquiry-based teaching is often discussed as a context that can support the development of CT, because it emphasizes students' active exploration of problems, question generation, and justification of solutions rather than only following given procedures. In mathematics, inquiry activities typically involve analyzing information, formulating hypotheses, questioning assumptions, comparing strategies, and evaluating the plausibility of solutions. These practices overlap with commonly cited CT processes such as interpretation, analysis, evaluation, inference, and explanation (Sikko & Ding, 2022; Sutiani, 2021).

In inquiry-based classrooms, instructors are often described as guides who support students' thinking through questioning and reflection(Bada & Olusegun, 2015). Classroom structures such as group discussion, inquiry tasks, and peer feedback may provide opportunities for students to articulate reasoning, consider alternative viewpoints, and critique arguments—activities that are frequently associated with CT in prior research (Herman & Pinard, 2015; Roslan et al., 2023). In this way, inquiry-based teaching helps students cultivate reflective and analytical thinking processes, enabling them to make well-reasoned decisions using their cognitive skills (Sikko & Ding, 2022; Sutiani, 2021).

However, its effectiveness remains a topic of debate among scholars, particularly regarding the appropriate level of teacher guidance and the strategies used to foster deeper learning and cognitive development (Almulla, 2023; P. Y. Chen et al., 2022). Rasmussen and Kwon (2007) criticized the inquiry approach in mathematics, noting that it emphasizes students doing mathematics by participating in mathematical discussions, posing conjectures, and solving new or unfamiliar problems; it tends to overlook the critical role of the instructor in guiding, structuring, and supporting the inquiry process. Without adequate support, students may struggle to engage effectively in problem-solving and reasoning, potentially leading to frustration or superficial understanding rather than deep thinking (Khalaf & Mohammed Zin, 2018).

The 5E instructional model (Engage, Explore, Explain, Elaborate, and Evaluate) has been widely used as an inquiry framework to structure classroom instruction (Gholam, 2019). Rather than prescribing specific content, the model provides a sequenced set of inquiry phases that can support teachers in enacting IBI in practice. In this sense, 5E is often described as a tangible and pragmatic scaffold for translating constructivist principles into classroom routines, helping teachers plan how students are introduced to a problem, guided to explore and generate ideas, supported to articulate and refine explanations, and encouraged to apply and extend their understanding (Bybee, 1997). Prior research suggests that the 5E model can support learning by connecting students' prior knowledge with new concepts and by promoting deeper conceptual understanding through structured inquiry experiences (Ergin, Kanli, & Ünsal, 2008). In addition, studies have reported that lessons designed around the 5E sequence may provide opportunities for students to engage in higher-order thinking processes, such as

explaining and justifying ideas, comparing strategies, and reflecting on their reasoning (Boddy, Watson, & Aubusson, 2003).

In addition, explicit CT instruction remains underdeveloped in higher education practice. Bellaera, Weinstein-Jones, Ilie, and Baker (2021), based on a survey of 176 humanities and social science instructors from universities in the United Kingdom and the United States, reported that instructors tended to rely more on implicit than explicit approaches to teaching CT. Their findings indicate that although CT is valued, instructors may lack sufficiently systematic and practical frameworks for explicitly teaching it. The problem is particularly visible in university mathematics education. Wang and Abdullah (2024) reviewed 15 empirical studies and found that only 27% explicitly stated CT development as a goal of mathematics courses. Even in those cases, the studies did not clearly describe how CT was incorporated into instructional design. Therefore, the literature reveals a shortage of explicit CT interventions in mathematics and practical guidance on how to embed such instruction systematically in mathematics classrooms.

To fill this gap, this study incorporates an explicit CT instruction into IBI using the 5E teaching model within a mathematics course for engineering students. The purpose was to investigate the effectiveness of this integrated approach in promoting students' CT development within the context of mathematics learning. The following research questions guided the study:

1. How can explicit CT instruction be effectively embedded in inquiry-based mathematics teaching?
2. To what extent does this instructional approach enhance students' CT skills and mathematics learning outcomes?
3. What are the changes in students' perception of CT?
4. Does this intervention affect students' CT in everyday life?

METHOD

In this study, interventions were implemented in an Ordinary Differential Equations (ODEs) unit within a higher mathematics course in a Chinese university. A 12-week quasi-experiment was conducted with 98 first-year engineering students to evaluate the intervention's effectiveness. They were randomly divided into three groups: a control group using traditional teaching methods, and two experimental groups adopting traditional IBI and the teaching method explicitly integrating CT into IBI (CT-IBI). We examined baseline comparability across groups using pre-test and found no significant differences in baseline achievement in CT and ODEs in the analysis.

This study emphasizes that instructor guidance plays a central role in supporting students' transition from intuitive reasoning to more formal and refined mathematical understanding (Laursen & Rasmussen, 2019). As Kuster et al. (2018) noted, effective IBL requires instructors to create opportunities for students to engage actively in doing, thinking, and discussing mathematics. This involves carefully selecting key mathematical ideas and designing tasks that allow students to "reinvent" mathematics

through guided inquiry and collaborative reasoning. Accordingly, this study proposes a new instructional framework that integrates the 5E model as a structural scaffold, with instructor guidance serving as the pedagogical core, thereby enhancing the feasibility and effectiveness of implementing inquiry-based teaching in the classroom. The IBI and CT-IBI instructions share the same teaching inquiry stages.

The CT-IBI explicitly integrated CT into learning objectives, teaching activities, and the evaluation stage of instruction (Al-Ghadouni, 2021) (Figure 1). This study adopts Facione (1990)'s CT skills—interpretation, analysis, evaluation, inference, and explanation—as both developmental targets and key metrics for measuring students' progress. The CT rubric was designed as the CT skills criteria. Before the intervention, students were introduced to the definitions, principles, and criteria of CT, as well as its instructional objectives and assessment goals, to encourage active application and practice of CT skills throughout the course.

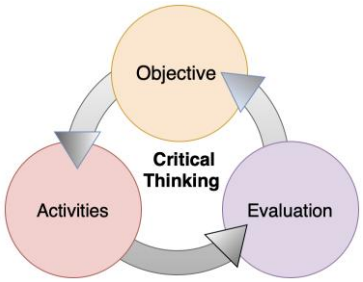

Figure 1
The CT-IBI explicitly integrated CT into learning objectives, teaching activities, and the evaluation stage of instruction (Al-Ghadouni, 2021)

The inquiry tasks were broken down into subject content questions and fire-up CT questions, forming sequential and heuristic small tasks that provided students with scaffolding to help them explore deeply (Habre, 2020; Kuster, Johnson, Rupnow, & Wilhelm, 2019; Bron & Prudente, 2025). There are 14 cognitive activities adapted from Facione (2011) that align with the context of mathematics subjects to guide the design of fire-up CT questions (Figure 2). Those questions encourage students to critically analyze assumptions, identify mathematical relationships, construct and articulate key concepts, evaluate their initial hypotheses and the validity of their solutions, and form final reasoning.

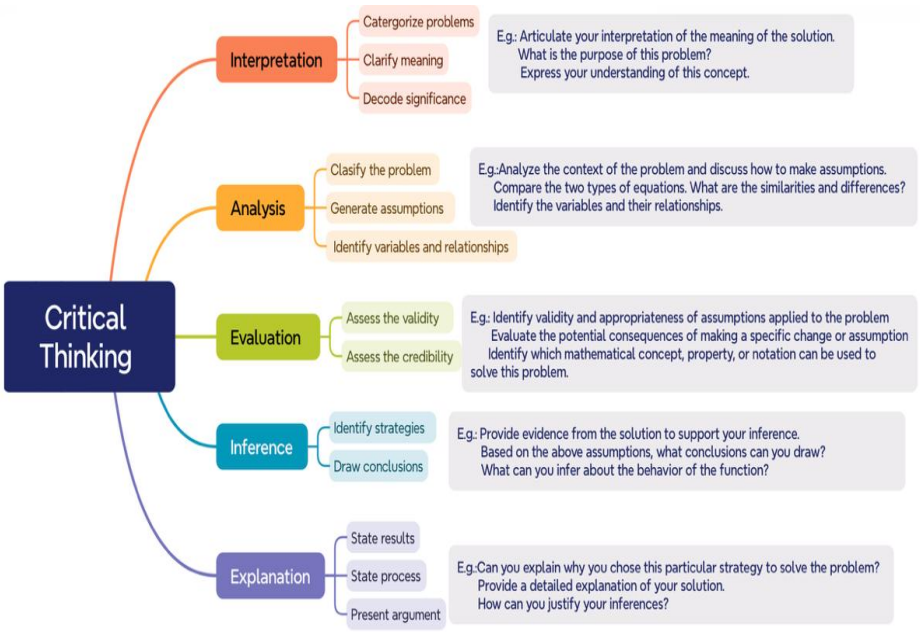

Figure 2

CT cognitive activities and example prompts in mathematics learning

A CT self-assessment form, designed based on a CT rubric. During the evaluation stage, the CT self-assessment tool was distributed to students through a teaching-assistive platform, allowing them to complete the assessment conveniently on their mobile devices. By engaging in the self-assessment, students were encouraged to critically evaluate their own thinking processes (Amelia et al. 2025).

In practice, the instructor provides guidance to support students' exploration and dialogue, helping them develop their conceptual understanding and reasoning based on the 5E steps. After students construct their initial interpretations and reasoning, the instructor facilitates a whole-class discussion where students present and share their work. Moreover, students' solutions and reasoning processes serve as valuable instructional resources for teachers to advance the mathematical learning agenda (Kuster, Johnson, Keene, & Andrews-Larson, 2018). The instructor collects students' solutions to refine their understanding and reasoning, ultimately fostering a deep, formal mathematical understanding within the learning community. The teaching activities in the 5E inquiry phase are presented in Table 1. Together, the CT infusion strategies and the 5E Inquiry phase formed the instructional design framework of CT-IBI (Figure 3)

Table 1
The teaching activities in the 5E inquiry phase

Phase	Process
Engagement	The instructor presents content questions connected with prior knowledge and new ODE concepts and CT fire-up questions within them. Students are asked to engage in group discussions to answer these questions. The process or results may lead students to discover new insights or generate a question, which stimulates their curiosity and interest to explore new types of ODEs equations.
Exploration	The instructor proposes inquiry tasks about new ODE concepts, theories, or knowledge aligned with the lesson's objectives. Those inquiry tasks consist of knowledge content and CT fire-up questions guiding students exploring effectively. Students are asked to reinvent knowledge by exploring those tasks in an interactive environment with peers and instructor.
Explanation	Students are provided with opportunities to demonstrate their understanding of new ODEs concepts, theory application, or procedures in the exploration phase. The instructor helps to clarify their misunderstanding and establish connections between those explanations and the experiences gained during the engagement and exploration phases. Presenting new concepts, theories, conjectures, and procedures to deepen their understanding, thereby refining their knowledge construction process. This process also provides students with opportunities to reflect their thinking processes.
Elaboration	The instructor designs application problems including ODEs concept problems and context-based problems. The problems were decomposed into smaller units embedded with CT fire-up questions, designed to activate cognitive engagement and facilitate the transfer of conceptual understanding to extended tasks and real-world problems. Students are asked to discuss in groups to solve these problems.
Evaluation	The instructor evaluates students' outcomes related to the lesson's objectives and process performance during discussion and presentation, encouraging them to evaluate and reflect on themselves. The CT self-evaluation form is sent to students for them to evaluate their CT processes.

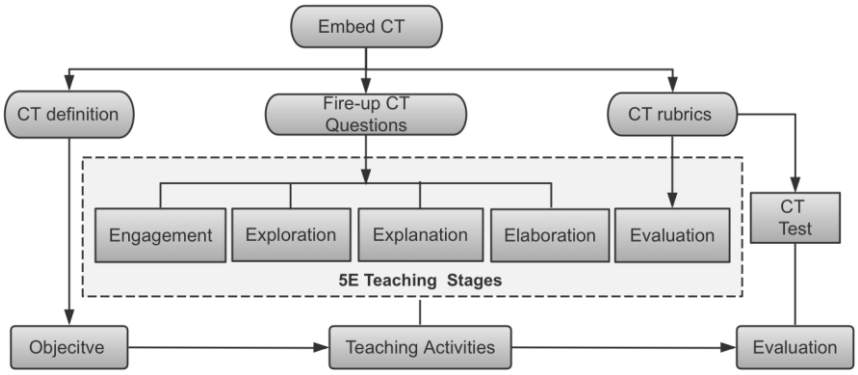

Figure 3
Instructional design framework of CT-IBI

FINDINGS

CT and ODEs Achievement

To examine the effectiveness of the CT-IBI intervention on students' CT and learning outcomes in ODEs, a post-test was administered after the intervention. The test items were based on authentic ODE problems. An expert panel was invited to evaluate the instrument's content validity. Clear scoring criteria were developed, and the raters were trained to minimize subjective interpretation. Inter-rater consistency and reliability were also considered. Finally, MANOVA was conducted to examine the effects of the intervention.. The Pearson correlation between CT and ODEs outcomes is r = 0.746. This indicates a strong linear relationship between the two variables, without evidence of multicollinearity. Figure 4 further supports this relationship by displaying scatterplot matrices for the three groups, each of which shows a consistent linear trend between CT and ODE scores.

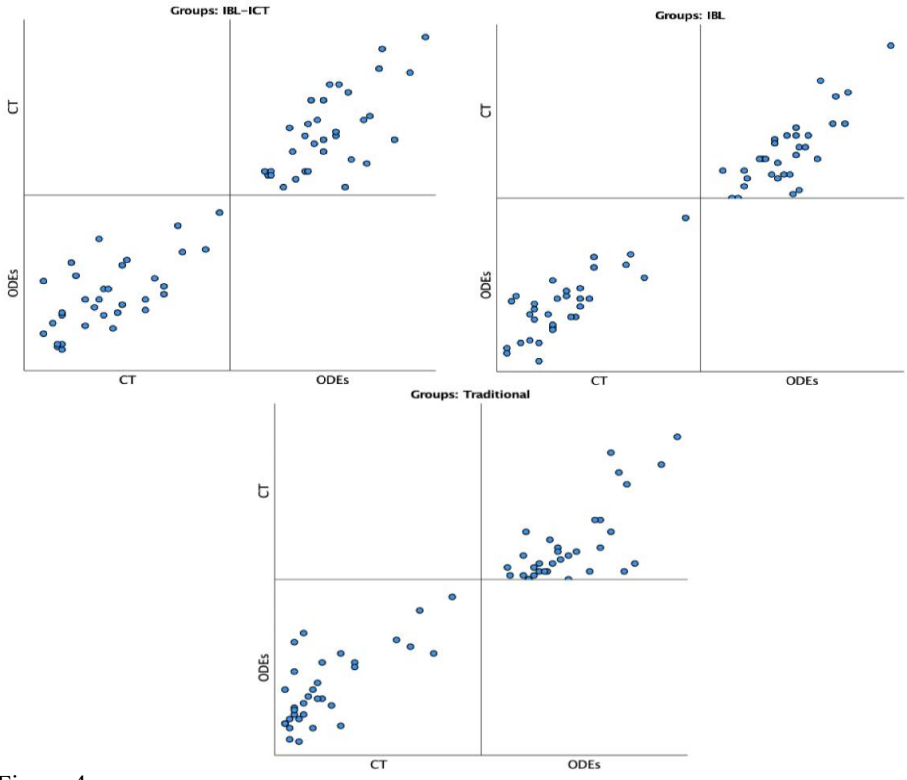

Figure 4
The scatterplot matrix of CT and ODEs learning outcomes

(a) CT Scores

The univariate ANOVA revealed a statistically significant effect of group on CT scores (*F(2,95) = 4.695, p<0.05).* Tukey's post hoc test was shown in Table 2, the results indicated that CT-IBI significantly outperformed the traditional group in CT (*p<0.05*). However, participants in the IBI and traditional groups didn't show a significant difference (*p>0.05*). Similarly, the CT-IBI group also didn't show a significant difference compared with the IBI group in CT (*p>0.05)*.

Table 2
Post Hoc test of CT among three groups

(I) Group	(J) Group	Mean Difference	Sig.	95% Lower	Upper
Traditional group	IBI	-1.684	0.357	-4.595	1.227
	CT-IBI group	-3.712*	0.008	-6.601	-0.824
IBI group	Traditional group	1.684	0.357	-1.227	4.595
	CT-IBI group	-2.028	0.226	-4.939	0.883
CT-IBI group	Control group	3.712*	0.008	0.824	6.601
	IBI group	2.028	0.226	-0.883	4.939

(b) ODEs Scores

The univariate ANOVA revealed a statistically significant effect of group on ODE scores (*F(2,95) = 3.432, p <0.05).* Tukey's post hoc test was shown in Table 3; the results indicated that the CT-IBI group significantly outperformed the traditional group in the ODE outcome (*p < 0.05*). However, participants in the IBI group and the traditional group didn't show a significant difference (*p<0.05*). Similarly, the CT-IBI group also didn't show a significant difference compared with the IBI group in ODE outcomes (p>0.05*)*.

Table 3
Post Hoc test of ODEs among three groups

(I) Group	(J) Group	Mean Difference	Sig.	95% Lower	Upper
Traditional group	IBI	-5.775	0.237	-14.2	2.651
	CT-IBI group	-9.091*	0.03	-17.451	-0.731
IBI group	Traditional group	5.775	0.237	-2.651	14.2
	CT-IBI group	-3.316	0.618	-11.742	5.109
CT-IBI group	Control group	9.091*	0.03	0.731	17.451
	IBI group	3.316	0.618	-5.109	11.742

Cognitive skills performance

The MANOVA analysis was used to evaluate the intervention's effectiveness on five dimensions of CT (Figure 5). The skills of interpretation, analysis, evaluation, and explanation showed significant differences among the three groups, whereas inference did not. Further, univariate ANOVA results indicated that the CT-IBI group showed a statistically significant difference compared with the traditional group ($p < 0.05$) in interpretation ($F=5.109$, $p<0.05$), analysis ($F=7.362$, $p<0.05$), evaluation ($F=5.139$,

$p<0.05$), and explanation skills ($F=5.995$, $p<0.05$). When comparing the IBI and traditional groups, a significant difference was found in interpretation skills ($F=5.109$, $p<0.05$).

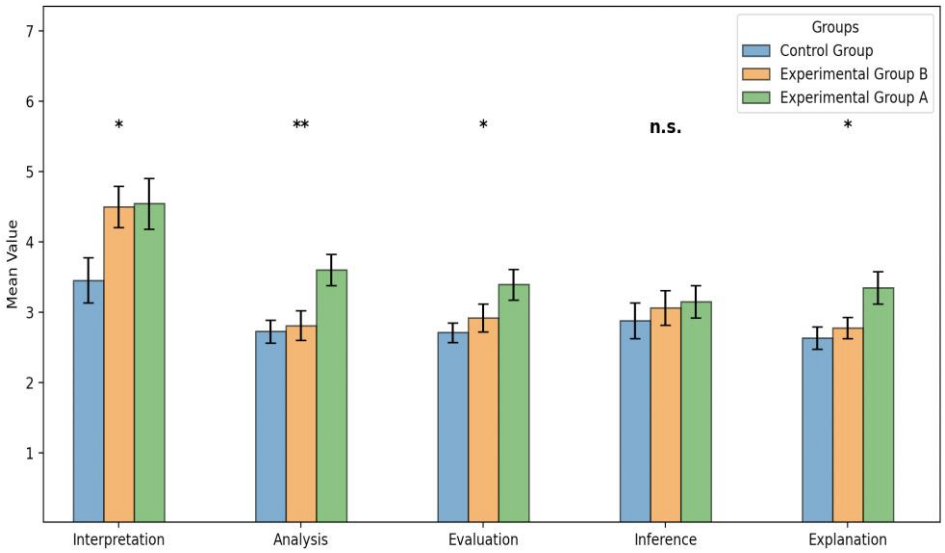

Figure 5
MANOVA result of post-test scores of cognitive skills in three groups

Interview

Semi-structured interviews were conducted with students after the intervention to explore how the teaching activities and instructional methods influenced the development of their CT skills. The interview data were analysed using thematic analysis. To ensure consistency in theme and category identification, two researchers independently coded the transcripts. Any coding discrepancies were discussed and resolved until agreement was achieved. A total of 9 students were selected from the sample of 98 students for the interviews, including two high-performing, two average-performing, and two low-performing students from CT-IBI group.

Firstly, the students' perception of CT showed that students in the CT-IBI group have a more consistent and systematic understanding of CT. Specifically, 8 out of 9 students in the CT-IBI group described CT-related cognitive skills or the standards of CT skills. Moreover, two students mentioned CT dispositions, such as self-reflection and questioning others' viewpoints. In contrast, students in the inquiry-based teaching group retained a fragmented and limited understanding of CT. More than half of the respondents in the inquiry-based teaching group (56%) mentioned cognitive skills related to CT, emphasizing its role in logical reasoning, problem-solving, and clear analysis. A significant portion (44%) associated CT with criticism and doubt, while a smaller percentage (22%) believed CT considers the problem from multiple perspectives or divides it into two. Additionally, 33% of students linked CT to

intellectual autonomy and independent thinking. This result aligns with (Chen, 2017), who suggests that Chinese students' understanding of CT is multifaceted and complex, possibly influenced by traditional cultural backgrounds. This indicates that establishing consistent definitions and standards of CT would be necessary to improve CT skills through instructional intervention.

Secondly, the researcher provided CT-IBI students with five key criteria for CT skills to encourage reflection on learning experiences where these cognitive processes were engaged or developed (The coding scheme is presented in Appendix B). The group discussion was the most frequently referred to by interviewees. The five cognitive processes of CT—interpretation, analysis, evaluation, inference, and explanation—occurred during group discussions. The findings revealed that peer interactions played a key role in enhancing students' understanding of ODEs. Students reported that group discussions enabled them to view problems from multiple perspectives, thereby promoting deeper analysis and more comprehensive mathematical reasoning. One student noted, "Discussions bring different perspectives on ODE problem-solving. By comparing others' methods with my own, I can find better solutions." Another added, "Group discussion gives us opportunities to evaluate others." These insights suggest that collaboration encouraged critical evaluation of their peers' and their own reasoning. Students also highlighted the role of independent thinking and explanation skills in group settings. As one student explained, "Thinking deeply and independently helps me better understand the ODE concepts and analyze the ODE problems." "Communicating with peers enhances my explanation skills," one student shared.

Furthermore, students noted that a progressive sequence of mathematical tasks guided them to experience deep understanding and reasoning processes, helping them to know how to explore rather than feel overwhelmed when confronted with a complex mathematical reasoning problem. One student reported that the step-by-step structure of the scaffolded tasks provided a clear line of thought and gradually guided them toward the correct direction of inquiry. The fire-up CT questions require them to explain concepts, make assumptions, choose suitable mathematical properties, and draw conclusions. In addition, students reported that the instructor's feedback clarified their misconceptions, deepened their understanding, and reinforced the rigor of their reasoning.

Interview data also indicated that students' awareness of the definition and criteria of CT positively influenced their motivation to enhance CT skills. Exposure to explicit CT instruction encouraged self-monitoring and reflection during problem-solving processes. One student noted, "The instructor taught us that the CT definition makes me aspire to improve my CT skills." Another shared, "When I express my ideas about ODE problems, I reflect on whether my explanation is clear and logical." Students also reported that CT standards served as useful guides for evaluating their reasoning. As one participant stated, "The standards of CT remind me to check whether I made a correct assumption and find all the variables' relationships during solving ODE problems."

In addition, a few students viewed self-assessment as a way to reflect more objectively on their reasoning processes and to become more familiar with CT skills. However, several students stated that self-assessment helped them reflect on the current lesson, but they did not have sufficient time to apply these reflections to improve their thinking during subsequent lessons

Thirdly, students perceived that the CT skills acquired in the mathematics classroom might be transferable to real-life contexts, as they understood the general definitions and principles of CT. A small portion of students reported that their familiarity with CT concepts and standards fostered a tendency to deep reflection and analytical thinking when engaging in everyday problem-solving:"When I see others commenting on trending events online, I don't just blindly believe what they say anymore. Instead, I start to think about whether their arguments make sense and whether the evidence they provide is actually reliable," a student replied. "I try to make my language as clear as possible and ensure that my arguments are supported by solid evidence. I think this is my progress", another student noted.

DISCUSSION

The findings indicate that both the IBI and CT-IBI groups performed significantly better than the traditional group in students' interpretation skills of CT. This result is consistent with previous studies showing that inquiry-based mathematics learning engages students in discussion, conjecture formation, justification of reasoning, and problem solving, all of which support higher-order thinking and deeper conceptual understanding (Agus Riawan & Agus Suyatna, 2019). Within IBI, students are encouraged to move from informal and intuitive reasoning toward more formal mathematical understanding, while instructors use students' responses to identify their thinking processes and provide guidance accordingly (Laursen & Rasmussen, 2019; Rasmussen & Kwon, 2007). In this way, feedback can align with the goals of each stage of inquiry, allowing the instructor to act as both a facilitator and a guide (Kuster et al., 2018).

However, the IBI group did not differ significantly from the traditional group in overall CT or ODE achievement. By contrast, the CT-IBI group significantly outperformed the traditional group on these outcomes. This suggests that inquiry alone may support some aspects of CT, but explicit integration of CT into instruction is more effective for improving broader CT performance and subject learning. In the CT-IBI approach, CT was deliberately embedded in learning objectives, classroom activities, and assessment tasks. Such structured and purposeful integration is consistent with Edwards (2017), who argued that CT should be intentionally incorporated into course design and instruction.

A possible explanation is that making CT an explicit instructional goal helps students recognize that CT is central to learning rather than an incidental by-product. Clearly stated goals provide students with direction and help them understand the purpose of classroom tasks (Calhoun et al., 2021). At the same time, CT standards offer concrete criteria that students can use to guide and monitor their own thinking (Paul & Elder, 2019a). When these standards are made visible, students are better able to analyse

peers' reasoning, identify weaknesses in their own arguments, and revise their responses accordingly. This may explain why the CT-IBI group performed better than the IBI group, in which CT was not explicitly taught.

Another important feature of the CT-IBI approach was the use of scaffolded inquiry tasks. Although inquiry tasks can be cognitively demanding, breaking them into smaller subtasks and combining them with content-focused prompts and CT fire-up questions appears to provide students with more accessible entry points into complex ODE problems. Such questioning may stimulate and sustain cognitive engagement during learning (Chikiwa & Schäfer, 2018; Facione, 2011). In addition, these structured tasks allow instructors to observe students' reasoning more closely, identify misconceptions, and provide targeted feedback. This supports Kuhn's (1999) argument that thinking skills should be taught as concrete, visible practices rather than treated as abstract educational ideals.

The findings also suggest that the self-evaluation form was less effective than other instructional strategies in promoting active self-regulation. Although several students in the CT-IBI group reported that the form helped them become more familiar with CT standards, most did not actively apply these criteria while solving problems. This finding aligns with Lämsä et al. (2023), who found that students tended to rely more on teacher-led assessment than on self-assessment, particularly when explicit support for self-assessment was limited. This implies that self-assessment tools alone may not be sufficient; students may require more modelling, guidance, and opportunities for reflective practice to use them effectively.

The quantitative findings further support the importance of CT in ODE learning. The Pearson correlation between CT scores and ODE achievement was $r = 0.746$ ($p < 0.01$), indicating a strong positive relationship. This suggests that students with higher CT levels tended to perform better in ODEs, reinforcing the close connection between CT and academic achievement in mathematics. The result supports the view that explicit CT instruction not only promotes thinking skills but may also enhance subject-specific learning outcomes.

Finally, the findings suggest that explicit CT instruction may also contribute to the development of students' disposition toward CT. Some students reported that participation in the CT-IBI programme encouraged them to think more carefully, seek evidence, and make more deliberate decisions in daily life. This is broadly consistent with Aizikovitsh-Udi and Cheng (2015), who found that systematic encouragement of CT in mathematics can foster both CT skills and disposition. At the same time, the transfer of such dispositions beyond the immediate learning context should be interpreted cautiously. As noted by Case (2005), the internalisation and sustained use of CT standards across contexts require long-term reinforcement. Therefore, further research is needed to examine how and under what conditions CT developed in mathematics can transfer to other domains and everyday situations

CONCLUSION

This study offers both a theoretical framework and a practical model for educators seeking to innovate curriculum design. However, several limitations should be noted. The sample was drawn from a single university, which may limit the generalizability of the findings. In addition, the study did not include a comparison group receiving traditional instruction with CT infusion, which restricts the ability to fully evaluate the effectiveness of the CTI-IBI approach. The absence of formal classroom observation tools also limited the collection of implementation data. While CT skill development was assessed quantitatively, students' dispositions toward CT were only explored qualitatively. Future studies should involve larger and more diverse samples and incorporate validated instruments to better evaluate the impact of instructional strategies on both CT skills and dispositions.

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