How Are Students' Prior Knowledge Differentiate Analytical Thinking Process in Identifying the Convergence of Real Number Sequences?

Agung Putra Wijaya  
Dr. candidate, Universitas Negeri Malang & Lecturer in Mathematics Education, Universitas Lampung, Indonesia, agung.putra.2003119@students.um.ac.id

Toto Nusantara  
Prof., Mathematics Department, Faculty of Mathematics and Science, Universitas Negeri Malang, Indonesia, toto.nusantara.fmipa@um.ac.id

Sudirman  
Dr., Mathematics Department, Faculty of Mathematics and Science, Universitas Negeri Malang, Indonesia, sudirman.fmipa@um.ac.id

Erry Hidayanto  
Dr., Mathematics Department, Faculty of Mathematics and Science, Universitas Negeri Malang, Indonesia, erry.hidayanto.fmipa@um.ac.id

Analytical thinking is needed in solving complex problems, including mathematics. One of them is identifying the convergence of real number sequences. The analytical thinking process includes three cognitive processes, namely differentiating, organizing, and attributing. It will allow a student to build knowledge by connecting experience with prior knowledge. Until now there has been no study that examines the analytical thinking process in terms of students' prior knowledge. This present study aims to describe students’ analytical thinking process in identifying the convergence of real number sequences in terms of students’ prior knowledge. This descriptive study with a qualitative approach involved twenty-one students as subjects consisting of students with low, medium, and high prior knowledge. Data were collected using tests and interviews. The triangulation method was carried out as a data validation process. Data analysis used the constant comparative method. The results showed that the higher the students' prior knowledge, the more detailed the differentiating process was. The organizing and attributing processes are carried out depending on the results obtained in the differentiating process. This is important to understand to be able to facilitate the development of analytical thinking skills by paying attention to students' prior knowledge.

Keywords: analytical thinking, prior knowledge, the convergence of real number sequences, thinking process, analytical thinking

INTRODUCTION
Analytical thinking is characterized by a process of differentiating information and organizing each piece of information to get a conclusion from a problem-solving (Art-in & Sitthipon, 2012). It includes the ability to distinguish and categorize the parts of a problem to be able to understand the important parts, how these parts can be related to each other, what causes and effects are related, and what are the reasons that underlie these relationships (Chonkaew et al., 2016; Anwar & Mumthas, 2014; Montaku, et al., 2012). Analytical thinking makes it easier for someone to view a problem (Aksu & Eser, 2020; Cigdemoglu & Geban, 2015; Kao, 2014) to get the right solution.

Analytical thinking includes three cognitive processes, namely differentiating, organizing, and attributing (Wirdiyatusyifa et al., 2021; Sanli, 2019; Ad’hiya & Laksono 2018; Sitorus & Masyarati, 2016; Paleeri, 2015). Differentiating is a process carried out to distinguish relevant and irrelevant or important and unimportant information from a problem. This process occurs when someone begins to sort out useful or useless information in problem-solving to be done. Organizing is a process carried out to manage relevant and important information to find a solution to a problem. In this process, one builds a systematic and coherent relationship between the pieces of information and begins to develop a problem-solving plan. Attributing is a process carried out to determine the focal point of the problem at hand. This process involves deconstruction in which one determines the purpose of the formation of the structure by these relevant and important parts.

To date, many studies have been conducted on analytical thinking. These studies can be classified into three major groups. First, the study on the development of instruments to measure analytical thinking skills by Akcakin & Kaya (2020), Aksu & Eser (2020), Ad’hiya & Laksono (2018), and Thaneerananon, et al. (2016). Second, the study related to the characteristics of an analytical thinker by Ferri (2010) and Shahbari (2020). Third, the study related to the dimensions of analytical thinking. It includes the cognitive process, profile, and characteristics of analytical thinking. Cognitive processes of analytical thinking include differentiating, organizing, and attributing (Anderson, et al., 2001; Kiong, et al., 2010). Regarding the analytical thinking profile, Husain, et al. (2020); Khusna (2020); Anggraini, et al. (2019); Putri, et al. (2019); Qolfathiriyus et al. (2018); and Irwanto, et al. (2017) have described students’ analytical thinking skills in solving problems. The characteristics of analytical thinking expressed by Parta (2016) include (1) pre-analytical, (2) partial-analytical, (3) semi-analytical, and (4) complete analytical.

The process of analytical thinking in solving problems depends on the prior knowledge it has. Weak prior knowledge becomes an obstacle to the analytical thinking process (Amintoko, 2017). The analytical thinking process will allow a student to build his knowledge by connecting experience with prior knowledge (Areesophonpichet, 2013). The new knowledge that a student has is the result of the integration of prior knowledge with new concepts obtained by a person (Amimah, et al. 2018). Prior knowledge influences students’ memory and understanding of learning (Wade & Kidd, 2019; Umanath & Marsh, 2014; Chen et al, 2014; Williams & Lombrozo, 2013; Arentz, et al,
However, until now there has been no study that explains the analytical thinking process in solving problems by paying attention to the prior knowledge that has been possessed.

Concerning solving problems, most of the topics in the Real Analysis course are abstract and complex. The convergence of the real number sequence is one of the topics in the Real Analysis so it requires analytical thinking in solving the problem (Lew, et al., 2016). This is one of the important topics to be mastered by students to understand further topics in Real Analysis, such as the sum of infinite series (Franci & Grammatico, 2022; Oehrtman, et al., 2014). Thus, the study that focuses on the topic of Real Number Sequences needs to be carried out continuously.

The purpose of this study was to describe students’ analytical thinking process in identifying the convergence of real number sequences in terms of prior knowledge. It will provide valuable information regarding the obstacles that students may experience in identifying the convergence of the real number sequence analytically by taking into account the students’ prior knowledge. This information can be used as capital in designing learning scenarios by taking into account the students' prior knowledge so that it can facilitate analytical thinking skills in solving problems, especially in identifying the convergence of real number sequences.

METHOD

Participants

This descriptive study with a qualitative approach was carried out with 21 third-year students who had taken the Real Analysis course at the Mathematics Education Study Program, Teacher Training and Education Faculty, Universitas Lampung as subjects. Before collecting the data, the researcher measured the students’ prior knowledge by giving a test. The prior knowledge test measures students’ understanding of the definition of the limit of a sequence (P1), the criteria for convergence (P2), the limit uniqueness theorem (P3), the criteria for divergence (P4), the unbounded sequences (P5), and the ratio test (P6).

Students' prior knowledge is divided into three categories, namely low, medium, and high prior knowledge. This category is determined by taking into account the mean ($\bar{x} = 67.11$) and standard deviation ($s = 4.89$) test scores. The number of students for each category of prior knowledge is presented in Table 1.
Table 1
The Number of students in each category of prior knowledge

<table>
<thead>
<tr>
<th>Prior Knowledge Test Score</th>
<th>Category</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>n ≤ 62.21</td>
<td>Low</td>
<td>7</td>
</tr>
<tr>
<td>62.22 &lt; n &lt; 72.01</td>
<td>Medium</td>
<td>9</td>
</tr>
<tr>
<td>n ≥ 72.01</td>
<td>High</td>
<td>5</td>
</tr>
</tbody>
</table>

After grouping students, the researcher analyzed the students’ prior knowledge. The description of prior knowledge for each category is presented in Table 2.

Table 2
Description of prior knowledge for each category

<table>
<thead>
<tr>
<th>Prior Knowledge</th>
<th>Category</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>P3</td>
<td>-</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>P4</td>
<td>-</td>
<td>-</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>P5</td>
<td>-</td>
<td>-</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>P6</td>
<td>-</td>
<td>-</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Data Collection
Data were collected through tests and interviews. The test of identifying the convergence of the real number sequences is presented in Figure 1.

Original version:
Diberikan barisan \( (x_n) \) yang didefinisikan dengan \( x_n = c^n \) dengan \( c \in \mathbb{R} \).
Selidiki konvergensi barisan \( (x_n) \).
Jika konvergen, tentukan limitnya.

Translate version:
Given a sequence \( x = (x_n) \) defined by \( x_n = c^n \) for \( c \in \mathbb{R} \) investigate the convergence of the sequence \( x = (x_n) \).
If it converges, determine the limit.

Figure 1
The test of identifying convergence of real number sequences

The problem structure and position of prior knowledge (P1 – P6) required in identifying the convergence of the real number sequence are presented in Figure 2.
After finishing the test, the subject was interviewed. The interviews conducted were semi-structured interviews to confirm the answers written by the subject on the answer sheet.

**Data Analysis**

Before performing the analysis, data validation is done through the triangulation method. The researcher compared the answer sheet and interview data. Then, data analysis was carried out using the constant comparative method (Kolb, 2012). It is done by comparing one data with other data constantly and then comparing categories with other categories regularly.

The results of this analysis are used to describe the analytical thinking process in each category of prior knowledge. It is described by paying attention to the differentiating, organizing, and attributing processes. The indicators of each cognitive process are presented in Table 3.
Table 3
Indicators of each cognitive process on analytical thinking

<table>
<thead>
<tr>
<th>Cognitive Process</th>
<th>Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differentiating</td>
<td>Determine the values of $c \in \mathbb{R}$ that allow to identify the convergence of the sequence $x_n = c^n$, namely $c = 1, -1 &lt; c &lt; 1, c = -1, c &lt; -1$ and $c &gt; 1$.</td>
</tr>
</tbody>
</table>
| Organizing        | 1. Determine the definition or theorem used to identify the convergence of the sequence $x_n = c^n$ at the specified values of $c$.  
2. Apply a definition or theorem to identify the convergence of a sequence $x_n = c^n$ at the specified values of $c$. |
| Attributing       | Conclude the convergence of the sequence $x_n = c^n$ at the specified values of $c$. |

FINDINGS

The analytical thinking process of students varies depending on the prior knowledge they have. They are presented in Table 4.

Table 4
The analytical thinking process in terms of prior knowledge

<table>
<thead>
<tr>
<th>Category of Prior Knowledge</th>
<th>Analytical Thinking Process</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Differentiating</td>
</tr>
<tr>
<td>Low</td>
<td>Not detailed</td>
</tr>
<tr>
<td>Medium</td>
<td>Semi-detailed</td>
</tr>
<tr>
<td>High</td>
<td>Detailed</td>
</tr>
</tbody>
</table>

The following illustrates students’ analytical thinking process in each category of prior knowledge.

Students with low prior knowledge

Students with low prior knowledge only understand the definition of sequence limit (P1) and convergence criteria (P2). In the differentiating process, they were only able to break down the problem into two of the four cases it should have been. They do the differentiating process in detail. With P1, they should be able to detect the appearance of a third case. Only, they didn’t. Furthermore, the organizing process is only carried out on cases that are found. They are not able to organize information on the problem and are limited to cases that have been found. This process ends with attributing to each case. They can carry out the attributing process even though it is limited to the cases that have been found. Finally, they are only able to solve the problem in two cases. The analytical thinking process of students with low prior knowledge (indicated in yellow) is presented in Figure 3.
Figure 3
The analytical thinking process of students with low prior knowledge

**Students with medium prior knowledge**

Students with medium prior knowledge understand the definition of sequence limit (P1), convergence criteria (P2), limit singularity theorem (P3), and divergence criteria (P4). When doing the differentiating process, they were only able to break down the problem into three of the four cases that should have been done. They managed to find a third case that was not found by students with low prior knowledge (semi-detailed). The organizing process occurs based on cases that have been formed. This process ends with attributing to each case. They can carry out the attributing process even though it is limited to the cases that have been found. They are only able to solve the problem in three cases. The analytical thinking process of students with medium prior knowledge (indicated in yellow) is presented in Figure 4.

Figure 4
The analytical thinking process of students with medium prior knowledge
Students with high prior knowledge

Students with high prior knowledge understand the definition of sequence limit (P1), convergence criteria (P2), limit singularity theorem (P3), divergence criteria (P4), infinite sequence (P5), and ratio test (P6). They were able to break down the problem into four complete cases as a result of the differentiating process (detail). The organizing and attributing processes were also successfully carried out in each case. They can solve the problem completely. The analytical thinking process of students with high prior knowledge (indicated in yellow) is presented in Figure 5.

![Analytical thinking process of students with high prior knowledge](image)

DISCUSSION

Students with low prior knowledge

By understanding the definition of sequence limit (P1), students are only able to break down the problem into two cases, namely Case 1 for $c = 1$ and Case 2 for $-1 < c < 1$. Ideally, students should be able to sort information into four cases. Students are only able to carry out the organizing process armed with prior knowledge about the definition of the limit of a sequence (P1). By understanding the convergence criteria of the sequence (P2), students can carry out the attributing process by concluding that in Case 1 for $c = 1$, the sequence converges to 1 and in Case 2 for $-1 < c < 1$, the sequence converges to 0.

The example of a student’s work is presented in Figure 6.

Original version:

For $c = 1$ \[ \lim_{n \to \infty} c^n = 1 \]

For $-1 < c < 1$ \[ \lim_{n \to \infty} c^n = 0 \]

Translate version:

For $c = 1$ \[ \lim_{n \to \infty} c^n = 1 \]

For $-1 < c < 1$ \[ \lim_{n \to \infty} c^n = 0 \]

![Examples of work of students with low prior knowledge](image)
These results are supported by the interviews between the researcher (P) and students with low prior knowledge (M1) as follows.

P : Why did you break down the problem into two cases, namely Case 1 for $c = 1$ and Case 2 for $-1 < c < 1$?

M1 : Because if the limit is determined, only they make it converges.

P : What about the other values of $c$?

M1 : Maybe they are divergent, but I don't know how to show it.

The results of the interviews showed that the students did not understand the criteria for divergence. For this reason, students only break down the problem into two cases. The limited prior knowledge of students has an impact on the differentiating process carried out. This is following the opinion of Sanli (2019) and Paleeri (2015) who states that to be able to carry out the differentiating process well, students need a certain amount of knowledge. After carrying out the differentiating process, students carry out an organizing process that is limited to the information that has been sorted in the differentiating process. The organizing process carried out involves standard procedures. Finally, the attribute process is carried out by concluding the results obtained in the organizing process. This shows that the organizing and attributing processes depend on the results of the differentiating process.

**Students with medium prior knowledge**

Students can carry out the differentiating process by breaking down the problem into three cases, namely Case 1 for $c = 1$, Case 2 for $-1 < c < 1$, and Case 3 for $c = -1$. By understanding the singularity limit theorem (P3) and the divergence criterion (P4), students can carry out the organizing process in determining the convergence of a real number sequence for $c = -1$. Students perform the attributing process by concluding that the sequence converges to 1 for Case $c = 1$, the sequence converges to 0 for Case $-1 < c < 1$, and the sequence diverges to Case $c = -1$.

The example of a student’s work is presented in Figure 7.

The results of interviews between researchers (P) and students with medium prior knowledge (M2) are as follows.

P : Are there any other values of $c$?
How Are Students’ Prior Knowledge Differentiate Analytical …

The results of the interviews showed that the students are not able to identify the convergence of the real number sequences for \( c < -1 \) and \( c > 1 \). For this reason, students only break down the problem into three cases. When compared with students with low prior knowledge, students with medium prior knowledge can perform differentiating processes better. This is following the opinion of Sanli (2019) and Paleeri (2015) who states that the differentiating process can occur if students have some supporting knowledge. Because students are only able to sort information into three cases, the organizing process is limited to those three cases. The organizing process is better when compared to students with low knowledge. Furthermore, the attribute process is carried out by providing conclusions from the organizing process that has been carried out in the three cases. Students’ analytical thinking process also shows that the organizing and attributing processes depend on the results of the differentiating process.

Students with high prior knowledge

Students are able to carry out the differentiating process by sorting information into four cases, namely Case 1 for \( c = 1 \), Case 2 for \(-1 < c < 1\), Case 3 for \( c = -1 \), and Case 4 for \( c < -1 \) and \( c > 1 \). Students can carry out the organizing process in determining the convergence of real number sequences for \( c < -1 \) and \( c > 1 \) by using ratio tests and infinite sequences. Students succeeded in doing the attributing process by concluding the convergence of the sequence for each case.

The results of interviews between researchers (P) and students with high prior knowledge (M3) are as follows.

\[ P : \text{Why did you break down the problem into four cases?} \]
\[ M3 : \text{In my opinion, certain values of } c \text{ have a different convergence. Therefore, I break down the problem into four cases so that easy to determine the convergence.} \]

\[ P : \text{Is it possible that there are other } c \text{ values?} \]
\[ M3 : \text{It’s not there anymore, sir. If the values of } c \text{ in the four cases are combined, then they represent each real number.} \]

The results of the interviews showed that the students break down the problem into four cases to simplify the process of identifying the convergence of the real number sequence. When compared with the analytical thinking process of students with low and
medium prior knowledge, the results of the differentiating process carried out by students with high prior knowledge are more detailed. This is supported by having complete prior knowledge to solve problems (Wade & Kidd, 2019; Umanath & Marsh, 2014; Chen et al, 2014; Williams & Lombozo, 2013; Arentz, et al, 2013; Fazio, et al, 2013). The results of this study indicate that analytical thinking is needed when dealing with problems with high complexity (Art-in & Sithipon, 2012) in which the process of sorting information occurs. To make problem-solving easier, someone is required to carry out the process of sorting information into simpler parts (Chonkaew et al., 2016; Anwar & Mumthas, 2014; Montaku, et al., 2012).

CONCLUSIONS

Different levels of students' understanding of prior knowledge lead to differences in analytical thinking processes in identifying the convergence of real number sequences. The differentiating process depends on students' prior knowledge. Supporting by the understanding of prior knowledge, students carry out organizing and attributing processes that are limited to the results of the differentiating process.

The differences in students' analytical thinking processes in each category of prior knowledge are described as follows: (1) Students with low prior knowledge perform the differentiating process incompletely (not detailed), which is continued in the organizing and attributing processes which are limited to the results of the differentiating process; (2) Students with medium prior knowledge can carry out the differentiating process quite completely (semi-detailed) which is continued in the organizing and attributing processes; and (3) Students with high prior knowledge can complete (detailed) the differentiating process which is continued in the organizing and attributing processes.

The results of this study indicate that the higher the students' prior knowledge, the more detail the results obtained in the differentiating process. The organizing and attributing processes are carried out depending on the results obtained in the differentiating process. Finally, the higher the student's prior knowledge, the more complete the resulting solution in solving the problem.

The results of this study provide an opportunity to conduct further research related to the characteristics of the differentiating, organizing, and attributing processes carried out by students in solving problems. This needs to be done to further explore students' analytical thinking processes in solving problems.

REFERENCES


How Are Students’ Prior Knowledge Differentiate Analytical ...


