



## Formulation of Learning Goals by Mathematics Teachers

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Formulating learning goals is one of the most important parts of the planning process for mathematics teachers. These goals are organised around three elements: action verbs; reference to specific mathematical content; and consideration of context. In this article, based on the notions of knowledge dimensions, cognitive processes and contexts, we took two samples of mathematics teachers from two different countries. Using cluster analysis, we sought to identify profiles of mathematics teachers in the formulation of learning goals, and to establish whether the schema of curricular approaches allows us to explain the evidence. The results show that there are two profiles that the curricular approach schema does not consider, and that one approach can encompass different profiles.

Keywords: curricular approaches, learning goals, lesson planning, teacher education, teacher profiles

### INTRODUCTION

The formulation of learning goals is one of the most complex elements in the instructional design process (Gagne et al., 1994). The teacher must be clear about the goals he or she intends to achieve when planning, since school tasks alone are not sufficient to ensure student learning (Sullivan et al., 2012). The formulation of learning goals conditions the learning opportunities that are offered to students (Chen et al., 2009). Moreover, a teacher's interpretation of a learning goal only makes sense in practice when he/she specifies its cognitive complexity by choosing tasks or workshops that fit a specific group of learners in a specific context (Authors, 2009; Sharp et al, 2019).

Learning goals, for a particular topic in school mathematics, can be understood as hypotheses about the specific ideas that students need to learn in order to achieve higher-level objectives. When viewed as hypotheses, learning goals can be created, tested, and refined based on evidence collected during teaching (Hiebert et al., 2018). Boaler (2022) emphasizes that learning goals should transcend the mere transmission of content to foster creativity and conceptual understanding, which helps reduce math anxiety and promote a growth mindset in students. Similarly, Darling-Hammond et al. (2020) point out that findings from the science of learning should be translated into

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transferable and measurable goals capable of guiding teaching practice toward holistic development. This means that the goals must not only be aligned with curricular content, but also with higher-order cognitive processes—such as reasoning, problem-solving, and mathematical communication—and with transversal skills that prepare students for diverse contexts.

Learning goals can also be defined as those capacities, competencies, knowledge, skills, aptitudes, abilities, techniques, skills, habits, values and attitudes that, according to different instances of the curriculum, students are expected to achieve, acquire, develop and use. (Lupiáñez, 2009). Therefore, it is possible to distinguish, at least, three levels of learning goals that we will call upper level, middle level, and lower level. The upper level refers to long-term goals: those that are achieved after a long formative period. These are cross-cutting and common expectations for all mathematics topics. Examples of these expectations include mathematical reasoning and its three processes, as well as the 21st-century skills assessed by PISA (OECD, 2023). Middle-level goals are usually associated with specific mathematical topic and a specific grade level. It is common for them to be expressed in sentences that highlight what is most important for the student to learn in that topic. An example of these expectations are the goals for a curricular design, lesson, or homework. The lower level is also associated with mathematics topics but corresponds to the most basic knowledge and routine procedures that the student must learn or activate when a specific task is completed (i.e., identifying an even number, and joining points on a plane). The higher-level goals are usually very clearly distinguishable. However, the boundaries between the middle and lower levels are more blurred (Authors, 2018a). For the purposes of this study, we focus on middle level goals.

We conducted an empirical study in which we compared the learning goals formulated by two groups of teachers. In the first group, the participants were practicing high school and middle school teachers who were beginning a graduate training program in Colombia. We relied on their responses to a questionnaire about one of their most recent lessons. They answered a question about the learning goals they purported during the planning of a lesson on a specific mathematics topic chosen by each teacher. The second group was composed of primary school teachers in training in Spain who, based on two tasks on rational numbers, formulated learning goals. This second group is the same one that was studied by Castro-Rodríguez et al. (2022)

We organize this article into five sections. First, we situate the study in the context of the literature and develop a framework for learning goals. Next, we present the objectives of the study. In the third section, we present the methodology and, in the fourth, the results. Finally, we present a discussion section that includes the implications of the study.

## **LEARNING GOALS BY MATHEMATICS TEACHERS**

DeLong et al. (2005) state that mid-level learning expectations, or goals, are characterized by their specificity (something concrete and for a lesson), premeditation (planned by the teacher before the lesson), intentionality (concrete actions or tasks are established for their achievement), cognitive balance (not too general, like the higher

level, but also not too concrete, like the lower level of expectations) and compatibility (it is related to other goals of the subject or of the lesson).

The goals, in terms of Mousley (2004), have a “connected knowledge” character; that is, the teacher should expect students to establish connections between previous and new knowledge, mathematical ideas and their representations, and academic and real contexts. Along the same lines, Lupiáñez (2009) states that specific learning goals are organized around three components: the capacity or capacities that express performance and are identified by action verbs, the explicit reference to a specific mathematical content, and the consideration of an application context.

Aguayo-Amagada et al. (2018) propose that, in order to analyze each of the three components proposed by Lupiáñez (2009), three variables can be considered, respectively: ability, content and situations. For the variable of ability, one can consider mechanical learning (terms, conventions, skills), but also understanding (concepts, principles, procedures), or the functional application of this learning to a new situation in which it makes sense. For the variable of content, we can consider the conceptual structure (the relationships with other concepts), the representation systems (the different ways in which the concept can be represented) and the phenomenology (phenomena that give meaning to the concept). Finally, the situation variables, understood as the aspect of the individual’s world in which a mathematical concept is used, can be analyzed from some mathematical and non-mathematical contexts, or, if in greater detail, from the PISA framework (OECD, 2013) into four categories: personal, occupational, social or scientific.

On the other hand, for Krathwohl (2002), in a revision of the taxonomy proposed by Bloom (1956), learning goals are usually defined in terms of a subject content and a description of what is to be done with or towards that content. That is, goal statements usually consist of a noun (subject content) and a verb or verb phrase (cognitive processes). Therefore, learning goals can be analyzed from two dimensions: the knowledge that is put into play and the cognitive processes that underlie it. In the knowledge dimension, four categories are identified (Krathwohl, 2002):

- Factual knowledge: The basic elements that students must know to be acquainted with a discipline or solve problems in it.
- Conceptual knowledge: The interrelationships among the basic elements within a larger structure that enable them to function together.
- Procedural knowledge: How to do something; methods of inquiry, and criteria for using skills, algorithms, techniques, and methods.
- Metacognitive knowledge: Knowledge of cognition in general as well as awareness and knowledge of one’s own cognition.

In the dimension of cognitive processes, six categories are identified:

- Remember: Retrieve relevant knowledge from long-term memory.
- Understand: Determine the meaning of instructional messages, including oral, written, and graphic communication.
- Apply: Use a procedure in a given situation.

- Analyze: To break something down into its constituent parts and detect how the parts relate to each other and to an overall structure.
- Evaluate: Make judgments based on criteria and standards.
- Create: To unite elements to form a novel and coherent whole or to make an original product.

This taxonomy is a hierarchy in the sense that the six main categories of the cognitive process dimension are assumed to differ in complexity: remembering is less complex than understanding, which is less complex than applying, and so on.

Krogh and Grimeland (2019), in the framework of school mathematics, interpret the four categories of knowledge as follows: Factual Knowledge refers to terminology and conventions; Conceptual Knowledge consists of mathematical concepts, ideas, and models; Procedural Knowledge includes everything related to doing something — calculations, algorithms, and strategies; and Metacognitive Knowledge is knowledge about cognition.

The categories of the cognitive process dimension are interpreted as follows: Remembering refers to memorizing things (such as definitions, terminology, or the steps of an algorithm); Understanding corresponds to goals that include verbs such as to interpret, to exemplify, to classify, to infer, to compare, to learn, to know, and to explain; Applying refers to doing something of a non-algorithmic nature (solving a problem that has no obvious solution); Analyzing refers to breaking something down to see relationships; Evaluating is testing something based on given criteria (i.e., proving a theorem); and Creating means producing something new, such as a hypothesis or an argument.

In a complementary manner, Castro-Rodríguez et al. (2022) adopt the proposal of curricular approaches of Rico and Lupiáñez (2008) to characterize the specific goals of a mathematical task, according to priorities, intentions and purposes. They label these curricular approaches as instrumental, structural, functional and comprehensive. The instrumental approach focuses on the mastery and use of basic facts, skills, and concepts, interpreted as tools and techniques. In this approach, the priority lies in the mastery of techniques and algorithms. The goals express specific routines, behaviors, or skills to be achieved that must be observed in the behavior of learners. In the structural approach, knowledge consists of a structured system of rules and formalized concepts based on deduction. Priority is given to the mastery of relationships and properties. This approach considers learning goals from a cognitive point of view that gives priority to the acquisition of knowledge. The functional approach emphasizes knowledge with which real situations are modeled and is oriented to the resolution of problems and questions in different contexts. The purpose is to develop mathematical competence in a variety of contexts and to foster functional thinking. The learning goals are concretized in tangible achievements, skills to function in society and applicable to everyday life. Finally, in the comprehensive approach, knowledge is the result of independent intellectual activity and learning is based on creativity. The goals do not express concrete results or observable skills, they represent the development of reasoning and divergent thinking.

When comparing the different reference frameworks described in the literature, we consider that the categories of knowledge and cognitive processes described by Krogh and Grimeland (2019) contain and discriminate in greater detail the objectives than the capacity variables formulated by Aguayo-Amagada et al. (2018) and the four curricular approaches proposed by Castro-Rodríguez et al.. The instrumental approach can be described from factual and procedural knowledge, together with the mental process of remembering and framed especially in mathematical contexts. The structural approach involves factual, conceptual, and procedural knowledge and the cognitive processes of understanding and analyzing in mathematical contexts. The functional approach involves especially conceptual and procedural knowledge, together with the mental process of applying and analyzing, in mathematical and non-mathematical contexts. Finally, the comprehensive approach can be described based on conceptual and metacognitive knowledge and the mental processes of evaluating and creating, in mathematical and non-mathematical contexts.

In line with these ideas, in Figure 1 we present the framework for analyzing learning objective formulation based on three categories: content, context, and cognition.

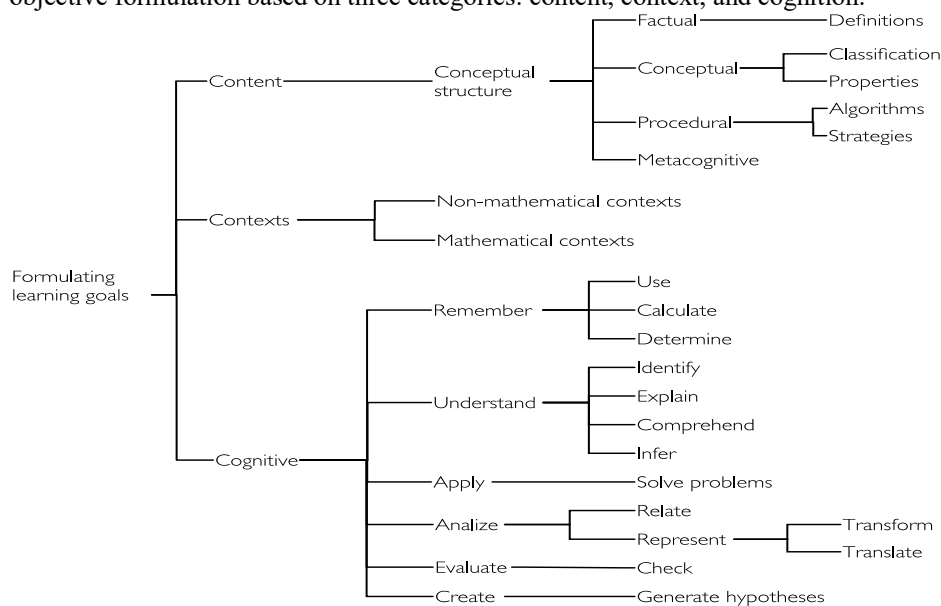


Figure 1  
Learning goals framework

Given that curricular approaches are also hierarchical, with a higher-level approach encompassing the lower-level ones, in Table 1, we summarize this description of curricular approaches based on the dimensions of content, context, and cognitive processes. For the content category, the variables are factual, conceptual, procedural, and metacognitive. For the context category, the variables are mathematical context and non-mathematical context. For the cognitive category, the variables are remembering, understanding, analyzing, applying, evaluating, and creating.

Table 1  
Description of curricular approaches

Variables/ Approach	Instruments	Structural	Functional	Comprehensive
Content category				
Factual	✓			
Conceptual		✓	✓	✓
Procedural	✓	✓	✓	
Metacognitive				✓
Context category				
Mathematical context	✓	✓	✓	✓
Non-mathematical context			✓	✓
Cognitive category				
Remember	✓			
Understand		✓		
Analyze		✓	✓	
Apply			✓	
Evaluate				✓
Create				✓

## RESEARCH OBJECTIVES

Based on the notions of knowledge dimensions, cognitive processes, and contexts, in this article, we seek (a) to identify profiles of mathematics teachers in the formulation of learning goals and (b) to establish, based on the resulting profiles, to what extent the schema of curricular approaches proposed by Castro-Rodríguez et al. (2022) allows us to explain the evidence.

## METHOD

To address the objectives, we took two samples. The first sample corresponds to a group of practicing teachers of Colombia who responded to a questionnaire asking about the learning goals they formulated for a recent lesson. The second sample was composed of trainee teachers of Spain who participated in the study by Castro-Rodríguez et al. (2022) and who formulated learning goals for specific tasks. We contrasted the learning goals of the two samples with the frame of reference that emerged from the literature (Figure 1), assigned the respective codes to each text segment and performed a cluster analysis that allowed us to identify similarities in the learning goals formulated. Next, we present the context and the description of the sample of teachers in the first sample. Subsequently, we describe the phases of data collection, coding, and analysis for this sample. Finally, we characterize the second sample.

### Context and characterization of the first sample of participants

The participants in the first sample were in-service mathematics teachers who had just started a graduate training program. The training program covers the planning, implementation, and evaluation of mathematics curricular designs. In this program, teachers are expected to develop in-depth pedagogical knowledge of mathematical

content oriented to support their decisions for lesson planning, implementation and assessment (Authors, 2018b).

The sample consisted of 92 mathematics teachers from official and private secondary schools in Bogotá, Cundinamarca, and Tolima (Colombia). Slightly more than half of them (52%) were male, most (85%) worked in official schools and 86% had more than five years of experience. Of the 92 teachers who responded to the questionnaire, 80 reported that they formulated learning goals for the lesson, and in some cases, that they formulated more than one goal for that same lesson. Therefore, this was a convenience sample of teachers interested in improving their teaching practices. It is important to mention that, in the Colombian context, teachers have curricular autonomy and are therefore not subject to a mandatory national curriculum.

#### **Collection of data from the first sample of participants**

A questionnaire was designed that, based on the curriculum model proposed in the program, could be used to describe, and characterize their classroom planning practices. Data were collected for all teachers who participated in the program between 2014 and 2018. The questionnaire was answered at the beginning of the program, to prevent the program from influencing teachers' responses. We designed a questionnaire in which teachers reported what they did during the development of a recent lesson. The selection of the topic and student group was at the teacher's discretion. Since this was a lesson that had just been implemented and we did not ask teachers for their opinion but rather asked them to report on the actions they took during the planning and implementation of that lesson, the information can be considered reliable (Desimone, 2009; Ross et al., 2003). The questionnaire item we considered in this study was as follows.

*Did you decide what you wanted your students to learn about the topic (e.g., in terms of learning goals, accomplishments, expected performances, or similar questions)? If you answered yes to the previous question, what did you want them to learn?*

Teachers responded individually, without interaction with the researchers. They had a limited time of 60 minutes to answer a 40-questions questionnaire.

#### **Context and characterization of the second participant sample**

In the study by Castro-Rodríguez et al. (2022), 80 students working toward a degree in Primary Education at the University of Granada, Spain, participated in the study. It was a convenience sample of trainee teachers in the final stage of their university studies. The instrument used to collect the information was a questionnaire with two tasks, related to rational numbers, used as items. Participants were told that the tasks were taken from a sixth-grade primary school textbook. Participants were asked to describe the learning goals for primary school students performing the proposed task. The authors of this study provided us with the participants' responses.

#### **Coding and data analysis for the two samples**

A process of content analysis (Mayring, 2015) and the learning goals framework (Figure 1) guided the identification of those codes that we assigned to the responses. The teachers' responses were coded by text segments, i.e., by teacher sentences that refer to a learning goal. Thus, a response is coded with three codes: type of knowledge,

cognitive process, and contexts. For each text segment, we assigned 1 and 0 according to whether it corresponded to each code, respectively.

In the Content category, the Conceptual Structure was organized into four major codes: Factual, Conceptual, Procedural and Metacognitive. In Contexts, we proposed the codes Mathematical Contexts and Non-mathematical Contexts. When a teacher did not indicate a context, such as “Use the characteristics of the decimal system to write and read numbers and operate with them”, we coded it as Mathematical Context. We coded as non-mathematical Context the cases in which some reference to a context or problem solving was indicated, such as “Distinguish characteristics of polygons and recognize them in their environment”.

In the Cognitive category, we have the codes Remember, Understand, Apply, Analyze, Evaluate and Create. In the Remember code, we associate the goals that refer to verbs such as to use or to employ, to calculate, and to determine. This is the case of “to determine the greatest common divisor of several numbers” and “use the properties of basic operations on real numbers for the solution of various situations”. In the Understand code, we cover goals related to recognize, distinguish, identify, understand, explain, and infer. This is the case of goals such as “to distinguish characteristics of polygons and recognize them in their environment”, “to recognize perpendicularity to find and construct heights in triangles”, and “to explain whole numbers and solve operations, such as addition, subtraction, multiplication and division”. In the Apply code, we include goals related to solving situations and solving problems. This is the case of goals such as “to solve situations that relate fractions in an easy way by doing it graphically based on halves, fourths and eighths, among others”. The Analyze code includes goals that refer to associating and relating, and those referring to making transformations within the same representation system and translating between representation systems, including generalizing with an algebraic expression. This is the case of “formulating a mathematical expression for the sum of internal angles”, “... the representation of addition and subtraction on the number line” and “... angular measurement in radians and establishing equivalences with degrees”. In the Evaluate code, we included goals related to check or verify. Such is the case of “to verify that the factors determined are solutions to problems involving expressions of the form  $x^2+bx+c$ ”. Finally, the Create code refers to goals related to generating hypotheses. For this last code, no goals were identified in any of the samples.

Thus, a learning objective such as “Distinguish characteristics of polygons and recognize them in their environment” was coded with the following three codes. In the content category, the code Conceptual was used to refer to *characteristics of polygons*. In the context category, the code Non-mathematical was used to refer to *their environment*. And in the cognitive category, the code Analyze was used to refer to *distinguishing and recognizing*.

Once the learning goals were coded, we proceeded to perform a cluster analysis with the resulting data. The centroids are the eleven codes described above, for six clusters. We used a Minkowski distance of degree 2. The smallest distance to the centroids of variables determined the clusters. The clusters obtained allowed us to formulate profiles

of teachers with respect to learning goals. These profiles are described based on centroid values greater than or equal to 0.5.

## FINDINGS

In the following, we present the results for the two samples and compare them.

### Results for the first sample

For the first sample, we coded 97 learning goals and proceeded to perform a cluster analysis with the resulting data. We present, in Table 2, the results for the six clusters. Values in bold correspond to centroids greater than or equal to 0.5 in each cluster.

Table 2  
Analysis for six clusters for the first sample

Variables/ Clusters	1A	2A	3A	4A	5A	6A
Content category						
Factual	0.00	0.31	0.00	0.05	0.00	0.00
Conceptual	1.00	0.00	0.20	0.52	0.38	0.14
Procedural	0.00	0.75	1.00	0.67	0.63	1.00
Metacognitive	0.00	0.00	0.00	0.00	0.00	0.00
Context category						
Mathematical context	0.89	1.00	0.90	0.00	0.00	1.00
Non-mathematical context	0.11	0.03	0.00	0.95	0.88	0.00
Cognitive category						
Remember	0.05	0.94	0.00	0.00	1.00	0.00
Understand	0.58	0.03	1.00	0.14	0.00	0.14
Apply	0.11	0.00	0.10	0.90	0.38	0.71
Analyze	0.42	0.00	0.30	0.00	0.00	0.71
Evaluate	0.00	0.06	0.00	0.05	0.00	0.00
Create	0.00	0.00	0.00	0.00	0.00	0.00
Percentage of goals	20%	33%	10%	22%	7%	8%

\*Error of squared distances = 67.85

As can be observed, the first profile is characterized by those teachers whose goals are focused on conceptual knowledge, framed in mathematical contexts and especially in the cognitive process of understanding. This is the case of goals such as "... that students will identify the quadratic function, represent it correctly and analyze it according to its characteristics". The second profile is characterized by those goals whose knowledge is essentially procedural, framed in mathematical contexts and whose main action is to remember. An example of this profile of teachers is the case of "determining the greatest common divisor of several numbers". The third profile is characterized by goals referring to procedural knowledge, the verb to understand and framed in a mathematical context. This is the case of "... that students understand what the process of factoring is". The fourth profile refers to conceptual and procedural knowledge, in non-mathematical contexts and to the verb to apply. This is the case of "using percentages to solve everyday problems, such as increments, discounts, taxes". The fifth profile refers to procedural knowledge, non-mathematical context, and the verb to remember. This is the case of "the different concepts of measurement: length, capacity, mass, the tools used to make these measurements and their use". Finally, the

sixth profile is characterized by procedural knowledge, mathematical context and the verbs to apply and to analyze. This is the case of “Applies algorithms to solve additive and multiplicative operations with decimal numbers”. We observe that the profiles with the highest percentages of goals are the second (33%), the fourth (22%) and the first (20%).

### Results for the second sample

For the second sample, when coding the responses of the participating teachers in the study of Castro-Rodríguez et al. (2022), we obtained the results presented in Table 3.

Table 3  
Analysis for six clusters

Variables/ Clusters	1B	2B	3B	4B	5B	6B
<b>Content category</b>						
Factual	0.00	0.00	0.00	0.00	0.00	0.00
Conceptual	1.00	0.96	0.00	0.67	1.00	1.00
Procedural	0.00	0.08	1.00	1.00	0.00	0.33
Metacognitive	0.00	0.00	0.00	0.00	0.00	0.00
<b>Context category</b>						
Mathematical context	1.00	1.00	0.94	0.00	1.00	1.00
Non-mathematical context	0.00	0.00	0.06	1.00	0.00	0.00
<b>Cognitive category</b>						
Remember	0.00	0.04	1.00	0.00	0.05	1.00
Understand	0.00	1.00	0.00	0.00	0.00	0.00
Apply	0.75	0.00	0.00	1.00	0.00	0.00
Analyze	0.00	0.19	0.00	0.00	1.00	0.00
Evaluate	0.25	0.00	0.00	0.00	0.00	0.00
Create	0.00	0.00	0.00	0.00	0.00	0.00
Percentage of goals	6%	33%	22%	4%	29%	8%

\*Error of squared distances= 14.09

We observe that the first profile is characterized by a purely conceptual knowledge, in a mathematical context and centered on the verb to apply. This is the case of “that they correctly use fractions to solve different types of problems in everyday life”. The second profile is characterized by conceptual knowledge, in a mathematical context and centered on the verb to understand. This is the case of “... representing a number in the form of a drawing as well as knowing if they have correctly assimilated the concepts of fraction: numerator and denominator”. The third profile is characterized by a purely procedural knowledge, in a mathematical context and centered on the verb to remember. This is the case of “Knowing how to calculate proportions”. The fourth profile is characterized by conceptual and procedural knowledge, in a non-mathematical context and centered on the verb to apply. This is the case of “that students know how to differentiate the different quantities referred to fractions, know how to use them in real life and know how to fraction a quantity according to the fraction”. The fifth profile is characterized by a purely conceptual knowledge, in a mathematical context and centered on the verb to analyze; for example, “[...] that they learn equitable distributions, that they learn the meaning of fractions”. Finally, the sixth profile is characterized by purely conceptual knowledge, in a mathematical context and centered

on the verb to remember. This is the case of “they are expected to compare fractions, for which they must remember the concept of fraction and its graphic representation in each case. They also must remember the l.c.m. to equal the denominator of both fractions and which is the greater”. We observe that the profiles with the highest percentages of goals are the second (33%), the fifth (29%) and the third (22%).

### Samples' comparison

Comparing the profiles of both samples, we observe that profiles 1A and 2B, 2A and 3B and 4A and 4B are identical, respectively. Profiles 1A and 2B are characterized by a purely conceptual knowledge, in a mathematical context and centered on the verb to understand. Profiles 2A and 3B are characterized by a purely procedural knowledge, in a mathematical context and centered on the verb to remember. Profiles 4A and 4B are characterized by conceptual and procedural knowledge, in a non-mathematical context and centered on the verb to apply.

By relating the profiles to the curricular approaches, we can identify that the six profiles do not strictly fit the four curricular approaches. For example, in Table 4, we represent with an “✓” the variables highlighted for each profile of the first sample (see Table 2) and their relationship with the description of the curricular approaches.

Table 4  
Relationship between profiles and first sample curricular approaches

Variables/ Clusters	1A	2A	3A	4A	5A	6A
Curricular approaches	Structural	Instrumental	Structural	Functional		Functional
Percentage of goals	20%	33%	10%	22%	8%	8%
Content category						
Factual						
Conceptual	✓			✓		
Procedural		✓	✓	✓	✓	✓
Context category						
Mathematical context	✓	✓	✓			✓
Context no mathematician				✓	✓	
Cognitive category						
Remember		✓			✓	
Understand	✓		✓			
Apply				✓		✓
Analyze						✓

We observe that five profiles (1A, 2A, 3A, 4A and 6A) can be interpreted from some approach (Structural, Instrumental and Functional). We also note that we can associate two different profiles (1A and 3A) with the structural approach and two other profiles (4A and 6A) with the functional approach. The fifth profile (5A) does not correspond to any approach and represents 8% of the sample. This allows us to claim that the characterization of learning goals by curricular approaches does not completely represent the evidence of practicing teachers.

Similarly, in Table 5, we relate the profiles obtained in the second sample to the curricular approaches.

Table 5  
Relationship between profiles and curricular approaches of the second sample

Variables/ Clusters	1B	2B	3B	4B	5B	6B
Curricular approaches	Functional	Structural	Instruments	Functional	Functional	
Percentage of goals	6%	33%	22%	4%	29%	8%
Content category						
Factual						
Conceptual	✓	✓		✓	✓	✓
Procedural			✓	✓		
Context category						
Mathematical context	✓	✓	✓		✓	✓
Non-mathematical context				✓		
Cognitive category						
Remember			✓			✓
Understand		✓				
Apply	✓			✓		
Analyze					✓	

We observe that, with this sample, five profiles (1B, 2B, 3B, 4B and 5B) can be interpreted from one of the curricular approaches (Structural, Instrumental and Functional). We also note that we can associate three different profiles (1B, 4B, and 5B) with the functional approach. In this case, there is one profile that does not correspond to any approach (6B) and represents 8% of the sample. This allows us to claim that the characterization of learning goals by curricular approaches does not completely represent the evidence of teachers in training.

## DISCUSSION

The results obtained in the cluster analysis show that the characterization of learning goals formulated by teachers by profiles is more appropriate than doing it only by curricular approaches. By having more detailed variables of knowledge, context, and cognitive processes, we can provide a characterization that better fits the evidence. In the case of the first sample, 8% of the responses are in a teacher profile that refers to procedural knowledge, in a non-mathematical context and focused on remembering. In the second sample, also 8% of the responses correspond to a teacher profile characterized by purely conceptual knowledge, in a mathematical context and focused on remembering. These two profiles, besides being different from each other, do not correspond to any of the curricular approaches. Therefore, we have two profiles that complement the four curricular approaches proposed by Castro-Rodríguez et al. (2022) to characterize learning goals.

This finding aligns with the observations of Authors (2020), who note that objectives derived from general curricular guidelines need to be translated into more specific operational goals to capture the diversity of teaching practices. Similarly, Lupiáñez (2009) emphasizes that objectives must be linked to mathematical skills and competencies, which explains why different profiles can coexist within the same

curricular approach: each profile reflects a particular way of translating competencies into observable goals.

Additionally, in both samples, the evidence makes it clear that, although there are goals classified as either structural or functional curricular approaches, these may be of a different nature. For example, in the first sample there are two statistically different profiles that can be labeled as structural approach and two profiles that can be labeled as functional approach. In the case of the second sample, we have three statistically different profiles that can be labeled as functional approach. This shows that the profile approach allows a characterization of learning goals that is finer than the curricular approach: a significant proportion (30% in the first sample in the structural approach, 39% in the second sample in the functional approach) of the evidence belongs to one approach but represents different views of learning goals in terms of the profiles. This result was to be expected given that the scheme of curricular approaches aims to provide, in four categories, a general characterization of school mathematics and was not designed to characterize learning goals. In this sense, this scheme does not necessarily account for everything that a teacher may think about in his or her practice when formulating learning goals.

This result was predictable, given that the curricular approaches framework seeks to provide, in a few categories, a general characterization of school mathematics and was not designed to characterize learning objectives. In this sense, this framework does not necessarily encompass everything a teacher might consider in their practice when formulating learning objectives. As Boaler (2022) and Darling-Hammond et al. (2020) point out, objectives must transcend mere curricular classification and address both creativity and conceptual understanding, as well as the transferability and measurement of learning. The diversity of profiles identified in our analysis reflects precisely this complexity: teachers formulate goals that integrate cognitive processes, contexts, and types of knowledge in a richer way than traditional curricular categories allow.

Another interesting result was to identify that the samples coincide in three profiles. Therefore, the profiles identified allow us to conclude that teachers both in training and in practice have similarities in the formulation of goals, although with differences in the proportions of each sample. If we take the cognitive process as a reference, we can see that teachers in both samples refer to the same thing in the other two categories. For example, if the goal focuses on the cognitive process of remembering, they tend to do so in mathematical contexts and refer to conceptual and procedural knowledge.

## CONCLUSIONS

The formulation of learning goals conditions the learning opportunities that are offered to students (Chen et al., 2009). In this study, we sought, based on the formulation of learning goals that mathematics teachers make in their lesson planning, to identify teacher profiles that complement the curricular approaches described by Castro-Rodríguez et al. (2022). For this purpose, we took two samples, a first one of practicing teachers and another one of teachers in training, and we performed a cluster analysis for each of them. The results show that there are profiles that the curricular approaches do not consider and that the same approach can encompass statistically different profiles.

A possible limitation that can be associated with this study is the variety of the samples, in terms of the stimulus that generates the goal, the teachers' training and the national curriculum that frames them. Regarding the stimulus for the formulation of learning goals, where the first sample has as reference a recent lesson and the second sample has two specific tasks on rational numbers, it is important to note that the profiles that emerge from the coding identify the teacher's intentions regarding student learning, and this is independent of the stimulus. The differences found only refer to the level of depth of the content, but not to the teacher's intentions. In terms of teacher training, the evidence allows us to identify diverse trends in terms of the proportions identified: the trainee teachers in the first sample are concentrated in profiles 2A, with goals whose knowledge is essentially procedural, framed in mathematical contexts and whose main action is to remember, and profile 4A, which refers to conceptual and procedural knowledge, in non-mathematical contexts and to the verb to apply. The teachers in the second sample concentrate on profiles 2B, with goals characterized by conceptual knowledge, in a mathematical context and centered on the verb to understand, and 5B, which refers to purely conceptual knowledge, in a mathematical context and centered on the verb to analyze. Although the first sample is contextualized in an autonomous curriculum and the second sample is subject to a compulsory reference curriculum, the evidence shows that the teachers' profiles in the formulation of learning goals is varied in both samples. In both samples, we have at least three profiles with percentages between 20 and 33%. Therefore, the curriculum does not represent a major bias in the results.

These results have implications for teacher training programs and for future lines of research. On the one hand, they can guide the efforts of teacher training programs to make teachers aware of the implications for student learning that have diverse formulations of expectations, beyond a particular content or grade level. On the other hand, we propose, as a future line of research, to compare the formulation of learning goals that mathematics teachers have before and after participating in training programs and to be able to determine in which aspects differences are identified.

Finally, another possible line of research is to study how these teacher profiles in the formulation of learning objectives relate to the setting of objectives by students. As Sides and Cuevas (2020) point out, goal setting by students in their mathematics classes can influence self-efficacy and motivation. Therefore, this goal setting may be influenced by the way teachers formulate and communicate learning objectives for the class.

## REFERENCES

- Gómez, P., & González, M. J. (2009). Analyzing and selecting tasks for mathematics teaching: A heuristic. In S. Lerman & B. Davis (Eds.), *Mathematical Action & Structures of Noticing: Studies inspired by John Mason* (pp. 179-188). Sense Publishers. <http://funes.uniandes.edu.co/429/1/GomezP09-2922.PDF>
- Romero, I., Gómez, P., & Pinzón, A. (2018a). Compartir metas de aprendizaje como estrategia de evaluación formativa. Un caso con profesores de matemáticas. *Perfiles Educativos*, 40(162), 117-137. <http://funes.uniandes.edu.co/23241/>

- Gómez, P. (Ed.). (2018b). *Formación de profesores de matemáticas y práctica de aula: conceptos y técnicas curriculares* [Training of mathematics teacher and classroom practice: concepts and curricular techniques]. Universidad de los Andes.
- Velasco, C. y Gómez, P. (2020). *Objetivos específicos de la educación básica y lineamientos curriculares: una visión desde las matemáticas escolares*. Documento no publicado. Universidad de los Andes. Bogotá. <https://is.gd/objetivosEBasica>
- Aguiayo-Amagada, C. G., Flores, P., & Moreno, A. (2018). *Concepto de objetivo de una tarea matemática de futuros maestros* [Concept of the objective of a mathematical task for future teachers]. *Bolema: Boletim de Educação Matemática*, 32, 990-1011. <https://doi.org/10.1590/1980-4415v32n62a12>
- Bloom, B.S. (Ed.), Engelhart, M.D., Furst, E.J., Hill, W.H., & Krathwohl, D.R. (1956). *Taxonomy of educational objectives: The classification of educational goals. Handbook 1: Cognitive domain*. David McKay.
- Boaler, J. (2022). *Mathematical mindsets: Unleashing students' potential through creative mathematics, inspiring messages and innovative teaching*. John Wiley & Sons.
- Castro-Rodríguez, E., Ruiz-Hidalgo, J. F., Lupiáñez, J. L., Fernández-Plaza, J. A., Rico, L., Segovia, I., & Flores, P. (2022). Specific Mathematics Learning Objectives Expressed by Teachers in Training. *International Electronic Journal of Mathematics Education*, 17(2). <https://doi.org/10.29333/iejme/11670>
- Chen, J. C., Reys, B. J., & Reys, R. E. (2009). Analysis of the learning expectations related to grade 1–8 measurement in some countries. *International Journal of Science and Mathematics Education*, 7(5), 1013-1031. <https://doi.org/10.1007/s10763-008-9148-5>
- DeLong, M., Winter, D., & Yackel, C. (2005). *Student Learning Objectives and Mathematics Teaching*. *Primus*, 15(4), 226-258. <https://doi.org/10.1080/10511970508984119>
- Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational researcher*, 38(3), 181-199. <https://doi.org/10.3102/0013189X08331140>
- Gagne, R. M., Briggs, L. J., & Wager, W. W. (1994). *Principles of instructional design*. Holt Rinehart and Winston.
- Hiebert, J., Morris, A. K., & Spitzer, S. M. (2018). Diagnosing Learning Goals: An Often-Overlooked Teaching Competency. In T. Leuders, K. Philipp, & J. Leuders (Eds.), *Diagnostic Competence of Mathematics Teachers: Unpacking a Complex Construct in Teacher Education and Teacher Practice* (pp. 193-206). Springer International Publishing. [https://doi.org/10.1007/978-3-319-66327-2\\_10](https://doi.org/10.1007/978-3-319-66327-2_10)
- Krathwohl, D. R. (2002). A Revision of Bloom's Taxonomy: An Overview. *Theory into Practice*, 41(4), 212-218. [https://doi.org/10.1207/s15430421tip4104\\_2](https://doi.org/10.1207/s15430421tip4104_2)

Krogh, K., & Grimeland, Y. (2019). *Teachers' learning goals for a mathematics lesson. In Eleventh Congress of the European Society for Research in Mathematics Education*, (No. 4). Freudenthal Group; Freudenthal Institute; ERME.

Linda Darling-Hammond, Lisa Flook, Channa Cook-Harvey, Brigid Barron & David Osher (2020) Implications for educational practice of the science of learning and development, *Applied Developmental Science*, 24:2, 97-140, DOI: 10.1080/10888691.2018.1537791

Lupiañez, J. L. (2009). *Expectativas de aprendizaje y planificación curricular en un programa de formación inicial de profesores de matemáticas de secundaria* [ Learning expectations and curriculum planning in an initial training program for secondary school mathematics teachers] (L. Rico, Ed.) [doctorado]. Universidad de Granada. <http://funes.uniandes.edu.co/798/>

Mayring, P. (2015). Qualitative content analysis: theoretical background and procedures. In A. Bikner-Ahsbabs, C. Knipping, & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education. Examples of methodology and methods* (pp. 365-380). Springer. [https://doi.org/10.1007/978-94-017-9181-6\\_13](https://doi.org/10.1007/978-94-017-9181-6_13)

Mousley, J. (2004). An aspect of mathematical understanding: The notion of “connected knowing”. In M. J. Hoines & A. B. Fuglestad (Eds.), *Proceedings of the 28th International Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 377-384). Bergen University College.

OECD (2023), *PISA 2022 Assessment and Analytical Framework*, PISA, OECD Publishing, Paris, <https://doi.org/10.1787/dfe0bf9c-en>.

Rico, L., & Lupiañez, J. L. (2008). *Competencias matemáticas desde una perspectiva curricular* [Mathematical competencies from a curricular perspective]. Alianza Editorial. <https://goo.gl/HjgnR0>

Ross, J. A., McDougall, D., Hogaboam-Gray, A., & LeSage, A. (2003). A survey measuring elementary teachers' implementation of standards-based mathematics teaching. *Journal for Research in Mathematics Education*, 34(1), 344–363. <https://doi.org/10.2307/30034787>

Sides, J. D., & Cuevas, J. A. (2020). Effect of Goal Setting for Motivation, Self-Efficacy, and Performance in Elementary Mathematics. *International Journal of Instruction*, 13(4), 1-16. <https://doi.org/10.29333/iji.2020.1341a>

Sharp, L. A., Bonjour, g. L., & Cox, E. (2019). Implementing the Math Workshop Approach: An Examination of Perspectives among Elementary, Middle, and High School Teachers. *International Journal of Instruction*, 12(1), 69-82. <https://doi.org/10.29333/iji.2019.1215a>

Sullivan, P., Clarke, D., & Clarke, B. (2012). *Teaching with tasks for effective mathematics learning* (Vol. 104). Springer Verlag. <http://link.springer.com.ezproxy.uniandes.edu.co:8080/book/10.1007/978-1-4614-4681-1>