International Journal of Instruction *July 2024 ● Vol.17, No.3*
e-ISSN: 1308-1470 ● www.e-iji.net *p-ISSN: 1694-609X*
pp. 1-18

Article submission code: 20230820155008
Received: 20/08/2023
Revision: 23/12/2023
Accepted: 07/01/2024
OnlineFirst: 01/04/2024

Development of Mathematical Concepts through a Problem-based Approach in Grade 3 Primary School Pupils

Daniel Doz
University of Primorska, Faculty of Education, Slovenia, *daniel.doz@pef.upr.si*

Mara Cotič
University of Primorska, Faculty of Education, Slovenia, *mara.cotic@pef.upr.si*

Nastja Cotič
University of Primorska, Faculty of Education, Slovenia, *nastja.cotic@pef.upr.si*

This research aims to design and evaluate an experimental model of problem-based mathematics instruction by conducting an experiment with 3rd-grade students in practical school settings. The objective is to determine the impact of the experimental instruction model on students' achievements in mathematics knowledge, focusing on the first taxonomy level according to Gagne. The study utilizes an experimental pedagogical research approach within existing primary school classrooms. The experimental group (EG) experiences the problem-based instruction model, while the control group (CG) receives behaviorist instruction. The experiment spans 8 months and includes 240 3rd-grade students. Results reveal that the EG, despite a minor pre-test advantage, demonstrates superior post-test achievements in arithmetic, geometry, and logic when controlling for pre-test scores. This confirms the effectiveness of problem-based instruction in enhancing conceptual understanding.

Keywords: concepts, mathematics, problem-based approach, problem-solving, primary school

INTRODUCTION

Problem-based teaching is defined as a pathway to foster more creative forms of thinking, experiencing, and evaluating, which is also the foundation for problem-solving (Assapun & Thummaphan, 2023; Cotič & Zuljan, 2009; do Amaral et al., 2021). Therefore, it should not be viewed solely from a narrow methodological perspective as a teaching method (cf. Strmčnik, 2001; Trullàs et al., 2022), but rather as a way of enhancing students' problem-solving abilities (Kadir et al., 2016; Simamora et al., 2017; Simanjuntak et al., 2021). Research has highlighted that learning is more effective when we introduce problem-solving and incorporate problem-solving logic into it (Aidoo et al., 2016; Darhim et al., 2020; Kadir et al., 2016; Selcuk, 2010; Sunguer et al., 2006; Surur et al., 2020; Wilder, 2015), also in mathematics (Ali et al., 2010; Fatade et al., 2013; Merritt et al., 2017). Consequently, students who study

Citation: Doz, D., Cotič, M., & Cotič, N. (2024). Development of mathematical concepts through a problem-based approach in grade 3 primary school pupils. *International Journal of Instruction*, *17*(3), 1-18.

mathematics with a problem-based approach are expected to achieve higher attainments and develop a deeper understanding of mathematics (Roh, 2003).

Moreover, the challenge of understanding mathematics is closely tied to the way we conceptualize mathematical knowledge (Godino, 1996). This is particularly crucial because mathematical concepts represent abstract entitles that require thorough investigation and elaboration to be fully comprehended (Goldin & Shteingold, 2001). Additionally, research has revealed that many students still encounter difficulties in understanding mathematical concepts, resulting in lower performance in problem-solving tasks (Tambychik & Meerah, 2010). However, since concepts are a fundamental aspect of learning new mathematics (Rittle-Johnson, 2017), it is of utmost importance for students to acquire solid conceptual knowledge.

To achieve this objective, we propose a method for developing mathematical concepts through a problem-based approach. In this study, we present the results of an experiment aimed at assisting primary school pupils in constructing new knowledge within a problem-based learning environment. The findings suggest that implementing a problem-based teaching model in mathematics may have positive effects on the effective development of conceptual knowledge.

Theoretical Framework

Problem-based learning

Strmčnik (2001) classifies problem-based teaching as a teaching principle. We refer to the principle of problem-based teaching when the teaching is problem-oriented, meaning that problem-based approaches are present in all stages of the learning process (De Graaff & Kolmos, 2003; Hung et al., 2008). Problem-based teaching extends to the entire curriculum, encompassing all its content and process dimensions, while problem-solving has a narrower meaning and encompasses only a part of the learning activities (Klegeris & Hurren, 2011). However, the most direct expression of problem-based innovations is evident in problem-solving.

Problem-based teaching is oriented toward exploring problem situations as well as acquiring new knowledge and investigating and reflecting on one's own learning style (De Graaff & Kolmos, 2003; Hung et al., 2008; Kartono et al., 2019; Klegeris & Hurren, 2011). In the process, we enhance and acquire both content-related, process-related, and metacognitive knowledge (Downing et al., 2011). Problem-based teaching is characterized by incorporating a problem situation, which can be defined by the teacher or done collaboratively with the students (Hung et al., 2008). Through this, students acquire new knowledge through exploration, argumentation, verification, and taking positions, while being mentally active throughout. Thus, problem-based teaching largely balances the ratio between reproductive and creative knowledge acquisition (Strmčnik, 2001).

The problem situation itself is not enough to trigger the learning process, as the establishment of a subjective attitude is also necessary for the problem situation to become a problem. The establishment of a subjective attitude in problem-solving is strongly related to students' interests and experiences, where both the cognitive component of problem-solving and students' motivational engagement in the problem situation are essential (Strmčnik, 2001). In problem-solving, the student's prior knowledge and, particularly, their experiences with problem-solving are important, as a problem becomes interesting to a student when they realize that they lack the knowledge to solve it but believe they can still solve it (Condell et al., 2010; Dunbar, 2017). At the same time, they are aware that solving the problem will not be easy and will require additional effort.

Problem-oriented teaching encourages the active role of the learner, the acquisition of useful knowledge, as well as self-regulated learning (Van Gog et al., 2020). In this context, learning is based on an authentic, important, and relevant problem situation that stimulates curiosity and allows for the formation of new questions (Hardy III et al., 2017). In problem-solving, it is not just about finding a solution but primarily about how students understand the context and background of the problem, whether they can identify it, solve it, and think about the solution. The focus of problem-solving is on transformation and less on adaptation, emphasizing learning processes rather than learning outcomes.

Wood (2003) offers a perspective on problem-based teaching that analyzes both the role of the student and the role of the teacher, as both are involved in problem-based teaching. From the student's perspective, important advantages of problem-based teaching are:

- the active role of the student in problem-solving, which enables the understanding of acquired knowledge and the development of collaborative skills important for further learning,
- facilitating interdisciplinary connections and knowledge transfer between subjects,
- high motivation of students for problem formulation and solving,
- activating prior knowledge and acquiring connected knowledge and the development of skills (cognitive processes).

In this teaching approach, the teacher has a considerably different role compared to traditional teaching, as the teacher (Wood, 2003):

- enthuses,
- shows interest in the learning processes and is interested in students' suggestions,
- guides and teaches less,
- encourages conversations and discussions among students, participating only occasionally (perhaps when additional guidance or support is needed),
- allows for "productive" unrest,
- checks whether individual learning goals of students are appropriately planned (goals that students formulate themselves and are related to problem-solving, but are individual),
- encourages the use of various resources,

- continuously assesses learning achievements and provides effective feedback,
- creates good working conditions, a productive atmosphere, and a safe learning environment.

The effectiveness of problem-based teaching can be evaluated based on the teaching or guidance provided by the teacher, characteristics of the students and classroom interactions, individual learning goals of students, as well as the problem itself. It is therefore a rather complex teaching method that involves several interconnected factors of success (Pimta et al., 2009). Problem-based teaching is student-centered, assuming that students will explore, connect theory with practice, utilize experiences and knowledge in problem formulation, and find solutions (Savery, 2006). Problems related to authentic real-life situations are crucial for the effectiveness of problem-based teaching (Misnasanti et al., 2017).

Development of Mathematical Concepts through Problem Situations

Nature of Mathematical Concepts

Based on their development processes, we can distinguish two types of concepts: (a) theoretical or relational concepts, and (b) empirical concepts (Mitchelmore & White, 2004). Theoretical (relational) concepts arise and develop through activities, summarizing schemes of relations between objects, while the formation and development of empirical concepts are based on the observation of sensory perceptible properties of objects.

For most concepts, it is rare to have a purely one-sided formation and development in an individual's consciousness, where one would deal solely with purely theoretical concepts on one hand and purely empirical concepts on the other. The development of most concepts usually occurs in a way that both theoretical and empirical characteristics complement each other, although the basic nature of a concept also influences the way it is introduced and developed (Kokol-Voljč, 1996).

For mathematical concepts, it is evident that they are predominantly of a theoretical (relational) nature since their content represents relationships. However, we cannot unilaterally confine the development of mathematical concepts to an exclusively theoretical framework. In didactic-mathematical literature (Kokol-Voljč, 1996), numerous examples can be found that support the belief that the development of most mathematical concepts is influenced by both relational and empirical aspects. This is particularly evident in the formation of concepts at the primary school level (for example, round shape and color are empirical concepts, while circle and circumference are already relational concepts).

Model for Developing Mathematical Concepts through Problem Situations

The development of concepts involves the processes of abstraction and generalization (Mitchelmore, 2002). Among the well-known theories of concept development (cf. Driver & Easley, 1978) are those proposed by J. Piaget, J. S. Bruner, and Soviet psychologists (such as V. Davydov, and S. L. Rubinstein). In more recent times, several models have been developed worldwide based on these theories to facilitate the development of mathematical concepts. One such model was developed by Dörfler

(1991). His theory is both cognitive-psychological (providing a model for the development of certain cognitive schemas) and didactic (by structuring these schemas, the teaching process in mathematics can be organized when introducing new concepts). The schema for the development of mathematical concepts according to Dörfler (1991) is as follows:

- Problem situation setting (the initial situation with a motive; for a specific concept, there are often various possibilities for the initial situation).

- Analysis of the initial situation (identifying the elements of the situation, their interrelationships, and the activities/operations that can be performed with them).

- Implementation of activities/operations (execution of relationships). Attention is directed towards the connections and relationships between the elements of the activity that are established through the activity. At the primary school level, activities should always involve concrete physical models (hands-on activities with didactic materials).

- Description of activities/operations and relationships - initial schematization (visual, verbal, geometric, etc.). By symbolically describing the relationships, they are fixed. At this stage, the symbols have a descriptive character, marking the elements of the activity and their numerical or geometric properties (characteristics). This stage involves separating from the specific elements of the activity.

- Implementation of activities in various other situations, while maintaining the fundamental relationships. Invariance is reflected only in the symbols of the activity's elements and the preserved schema of relationships.

- Schematization of activities/operations and the identification of conditions for their execution. This stage represents a transition from activities with initial objects to operating with their symbolic representations. By symbolically describing the elements and relationships, we introduce a kind of search tool - for discovering activities that are "identical" in this sense.

- Further differentiation of the form of activities and relationships from the substantive meaning of their elements. With a new symbolization of the invariants of the activity (which again represents a reflection at the symbolic level), we replace the elements of the activity and their transformations with symbols. The symbols then take on the character of objects. The elements of the activity become formal objects, no longer determined by their content but only by the feasibility of operations with them. The resulting mental objects are "generalizations" (or concepts).

- Objectifying relationships and implementing activities of the next (higher) level with these "generalizations" (concepts constructed thus far). The feasibility of operations with the products of previous operations also requires clear descriptions of these products. Implementing activities/operations at higher levels leads to new symbolic descriptions, new means of representation, and new objects that allow for structurally similar activities.

- Concretizing the schemas (activities/operations in which the developed concept acts as an instrument - the application of the concept). This not only generates concepts derived from activities but also activities “derived from” formal concepts (thus, concepts lead to activities).

Looking at the presented schema, it is evident that the process of abstraction or generalization plays a crucial role in the development of a mathematical concept. By employing a process based on a concrete problem situation, we form the basic cognitive structure that represents the intended mathematical concept. Symbolization (drawing, sketching, schema, diagram, mathematical symbol) plays a significant role as it occurs whenever we transition from a lower to a higher level (cf. Bardini & Pierce, 2015).

It is important to emphasize that the steps mentioned in the construction of a mathematical concept are closely interconnected and intertwined. When developing various mathematical concepts, these steps are not always expressed to the same extent, and sometimes some steps are omitted. The steps of the schema should not be understood hierarchically (as phases that must necessarily follow each other in a specific order), but as aspects of a comprehensive process (Kokol-Voljč, 1996).

All the information mentioned above is summarized in Figure 1, where the nine previously mentioned points are organized into three main sections (levels) and the applications (cf. Cotič, 2009).

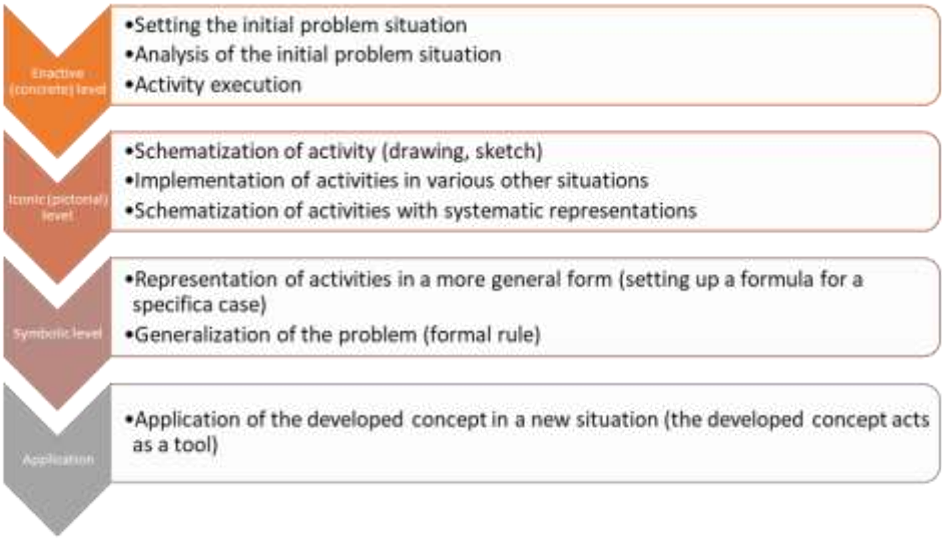

Figure 1
The model of levels and applications (cf. Cotič, 2009)

The stage of abstraction and generalization typically starts around the third grade (Mitchelmore & White, 2000; Novack et al., 2014). During this grade, students undergo a shift from fundamental skills and knowledge to more advanced cognitive abilities (de Koning et al., 2007; Ye et al., 2016). Third-grade students frequently develop basic mathematical skills, forming a foundational understanding. Implementing problem-based instruction at this juncture enables us to evaluate how these students expand on their current knowledge and apply it to more complex problems.

Considering Dörfler's (1991) model, students are immersed in exploring mathematics within familiar contexts, such as everyday shapes or real-life data analysis. They engage in hands-on activities, involving physical measurements, shape observation, counting, and object grouping. Symbolic representations are introduced progressively as students transition from the concrete-operational phase to abstract understanding. The problem encourages the application of the developed concepts in a creative and practical context, solidifying students' comprehension of the topics and relationships between objects. In the present research, we applied this model to problem-based teaching of mathematical concepts.

Additionally, the novelty of the study lies in the application of Dörfler's model in Grade 3 of primary schools, where problem-solving, although mandated by the national mathematics program (Žakelj et al., 2011), is rarely implemented. In this case, the traditional behaviorist teaching model is applied. Therefore, students generally solve exercises in the "drill-and-practice" manner, with immediate feedback provided by their teachers. Instruction is typically highly structured, implying that complex mathematical concepts are broken down into smaller components. Learning assumes a progressive, step-by-step objective, with each step building upon the previous. The behaviorist learning model is generally teacher-centered, as teachers guide the learning process, provide information, and direct students' activities. On the other hand, the proposed problem-based model centers around students engaging in authentic problem-solving activities; therefore, their learning is driven by the exploration and resolution of real-world problems and scenarios. Instead of focusing solely on skills and procedures, the problem-based model emphasizes the application of knowledge to solve complex problems, aiming to develop critical thinking and problem-solving skills.

This model is typically student-centered, as students take a more active role in their learning process. They are responsible for identifying problems, conducting investigations, and proposing solutions. The problem-based model aims for a deeper understanding of concepts. Instead of rote memorization of formulas and procedures, the focus is on understanding the underlying principles and applying them in novel situations.

Empirical Work

Aims of the research

The research problem is focused on designing and evaluating an experimental model of problem-based mathematics instruction. In practical school settings, we conducted an experiment with 3rd-grade students to determine whether the use of the experimental model of problem-based instruction had a statistically significant impact on students' achievements in mathematics knowledge. We particularly focused on the first taxonomy level according to Gagne. Our general research hypothesis is:

H: Students exposed to the experimental model of problem-based mathematics instruction (EG) will outperform students exposed to behaviorist mathematics instruction (CG) in solving mathematical problems related to basic and conceptual knowledge across all mathematical areas (arithmetic and algebra, geometry with measurement, and logic and language).

METHOD

The present study utilized the quasi-experimental method in pedagogical research because randomization is seldom feasible in educational research, given the challenges of randomly selecting students within classes (Green, 2010). The experiment was conducted within existing classrooms of primary schools. This means that before the experiment, there was no equating of the classes to eliminate random differences. The group of students exposed to the experimental factor was the experimental group (EG). The group of students not exposed to the experimental factor was the control group (CG). The experimental factor included the experimental model of problem-based mathematics instruction, which was developed based on foreign research while considering the Slovenian context. In both the experimental and control groups, primary school teachers were involved, and matched in terms of their educational qualifications.

In the present research, third-grade pupils were chosen because, in this grade, students undergo a transition from basic foundational skills and knowledge to more complex cognitive abilities (Ye et al., 2016). Therefore, studying students in these grades enables us to examine how problem-based instruction impacts their conceptual understanding during this pivotal stage. Additionally, third-grade pupils often acquire basic mathematical skills, establishing a foundational understanding. Introducing problem-based instruction at this point allows us to assess how these students build upon their existing knowledge and apply it to more complex problems.

Participants

The study was conducted on a sample of 240 students from the 3rd grades of four randomly selected coastal primary schools. Specifically, 100 students were included in the experimental group (57% females), while 140 students were in the control group (52% females). All selected primary schools were urban schools with comparable and satisfactory working conditions.

Among the 38 primary schools in the Coastal-Karst Slovenian region, four primary schools were selected randomly using a random number generator. Within the chosen schools, two were randomly assigned to the experimental group (EG), while the remaining two constituted the control group (CG). All Grade 3 students from each selected school were included in the EG and CG groups, respectively. Due to the absence of randomization, results should be interpreted with caution.

Ethical considerations

After receiving approval from school principals and teachers, all participants and their parents were informed about the research objectives, its duration, and potential benefits. Participation in the study was voluntary, and informed consent from participants' parents was obtained before the study commenced. Anonymity and confidentiality were ensured throughout the research process. The research adhered to the European Code of Conduct for Research Integrity (ALLEA, 2023) during its entirety.

Instruments

Within the framework of the empirical research approach, a pedagogical experiment was used. Data was gathered using knowledge tests (two knowledge tests on mathematical problems). For the purposes of the research, we designed two knowledge tests, a pre-test and a post-test.

Two knowledge tests were devised for research purposes. The initial and final mathematical knowledge of both the experimental and control groups were evaluated using corresponding initial and final knowledge tests. These tests were designed by incorporating Gagné's taxonomy (Cotič & Žakelj, 2004) and the mathematical content of third-grade curriculum (Žakelj et al., 2011).

Each test consisted of 7 tasks. Students were given one school hour (50 minutes) for each test. All tasks were open-ended. The tasks within the tests addressed the taxonomic levels concerning the understanding of concepts and definitions (conceptual knowledge). The test assessed students' knowledge in arithmetic (A), geometry and measurement (G), and logic and set theory (L). Arithmetic problems were emphasized due to their familiarity with early-grade students.

The properties of both knowledge tests (objectivity, reliability, and validity) were demonstrated using a pilot sample of 102 third-grade pupils from two randomly selected coastal schools.

The test's validity was ascertained through qualitative analysis, involving experts in mathematics education, mathematicians, and elementary school teachers. Their critical evaluation led to content and structural improvements. Additionally, the test scores showed a strong and statistically significant correlation with students' mathematics grades (r = .89; p < .001).

Test objectivity involved two aspects: (1) the impartiality of test administration, and (2) the neutrality of grading pupils' responses. In terms of the former, researchers provided comprehensive information about the test to minimize their influence on students. Regarding the latter, the test included numerous closed-type questions. Moreover, the tests were assessed by three independent individuals: a researcher (R) and two mathematics teachers (T1, T2). The inter-rater correlations were 1.00 (T1, T2), .99 (T2, R), and .99 (T1, R) as per Hallgren (2012).

To gauge instrument reliability, the parallel tests method was used (Mueller & Knapp, 2018). Students took two tests; the second was a parallel version of the first, assessing the same content and taxonomic levels. Reliability was determined by the correlation between the two results. This correlation was strong, positive, and statistically significant (r = .97; p < .001), signifying the instrument's excellent reliability.

Furthermore, the test and its items' difficulty were considered (Mahjabeen et al., 2017). Specifically, only items with a difficulty index (ratio of correct answers to total answers) between 10% ("difficult" questions) and 90% ("easy" questions) were retained.

Procedure and data analysis

The initial knowledge test was solved by both the control and experimental groups before the start of the experiment, and the final knowledge test was solved by them after the completion of the experiment under the same conditions and with the same examiner. The research lasted for 8 months.

During the research period, mathematics classes for the 3rd grade in the experimental group were conducted using the experimental model of problem-based teaching, while the control group used the behaviorist model.

Data in the empirical part were analyzed using the statistical analysis software SPSS 26.0. The following statistical procedures were employed:

- Means, standard deviations, minima, and maxima;
- Kolmogorov-Smirnov test for normality of distribution, which has shown that both scores on the pre-test and those on the post-test are normally distributed;
- Cochran-Cox's approximation method for t-test to determine differences between EG and CG;
- Analysis of covariance (ANCOVA) to determine differences in achievements considering results on the pre-test.

FINDINGS

Before starting the analyses, the normality of the data was assessed using the Kolmogorov-Smirnov test. The results indicated that both the pre-test achievements ($p > .05$) and post-test achievements ($p > .05$) followed a normal distribution. Consequently, parametric statistical tests were employed.

Pre-test achievements

Table 1 presents the descriptive statistics for the pre-test. The independent samples *t*-test revealed that students from the EG outperformed the CG with statistically significant differences in arithmetic ($t(238) = 3.14$; $p < .001$) and logic ($t(238) = 2.90$; $p = .002$). However, no statistically significant differences in achievements were observed regarding geometry ($t(238) = 0.069$; $p = .473$).

Table 1
Descriptive statistics for the pre-test

Topic	Group	M	SD	min	max
A	EG	1.43	.74	0	2
	CG	1.16	.75	0	2
G	EG	.46	.50	0	1
	CG	.46	.50	0	1
L	EG	.62	.63	0	2
	CG	.35	.61	0	2

Post-test achievements

Table 2 displays the descriptive statistics for the post-test. The ANCOVA analysis, incorporating a single covariate (the pre-test), revealed a significant influence of the learning type on the post-test score while adjusting for the pre-test in the case of arithmetic ($F(1,238) = 6.02$; $p = .015$), geometry ($F(1,238) = 4.01$; $p = .046$), and logic ($F(1,238) = 4.07$; $p = .045$). Hence, it is evident that the experimental group (EG) exhibited superior accomplishments in all mathematical subjects compared to the control group (CG).

Table 2
Descriptive statistics for the post-test

Topic	Group	M	SD	min	max
A	EG	2.03	.96	0	3
	CG	1.53	1.11	0	3
G	EG	1.10	.94	0	2
	CG	.85	.96	0	2
L	EG	.99	.88	0	2
	CG	.62	.82	0	2

DISCUSSION AND CONCLUSIONS

In today's world, the significance of procedural knowledge has diminished, while the demand for conceptual, theoretical, and problem-solving skills has substantially increased (Aydoğdu & Ayaz, 2008). The importance of complex knowledge, encompassing everything from foundational reading and computational skills to tackling intricate problems and approaches (English, 2008), is emphasized. Changes have occurred in how knowledge is perceived, transitioning from a notion of knowledge as singular and unchanging to complex and dynamic. The basis for understanding the types and aspects of knowledge stems from theories and classifications of knowledge. It is important for both elementary school teachers and mathematics educators to recognize the various aspects, types, and levels of knowledge, to be able to discern which to prioritize in different situations, and to know how to introduce, address, reinforce, and ultimately assess and evaluate them in mathematics instruction (Cotič, 2010). The goals of mathematical education, which previously concentrated on concrete content and procedural knowledge, are now increasingly complemented with conceptual and process-oriented knowledge, focusing on problem-solving strategies that are transferable to other subject areas and real-life situations (Chapman & Aspin, 2013; Cotič, 2010).

While problem-solving in mathematics is fundamental in contemporary mathematical instruction (Ali et al., 2010; Fatade et al., 2013; Merritt et al., 2017) due to the expectation of enhanced proficiency and deeper comprehension (Roh, 2003), conceptual understanding holds a pivotal role in mathematics education (Godino, 1996; Goldin & Shteingold, 2001). This significance arises as research indicates persistent difficulties students face with elementary mathematical concepts (Tambychik & Meerah, 2010). The primary objective of this study was to determine the optimal approach for introducing conceptual knowledge to primary school students: (1) the conventional (behavioristic) teaching and learning method in mathematics, or (2) a problem-based approach to teaching and learning (cf. Dörfler, 1991).

Both in the EG and in the CG, teachers conducted mathematics lessons according to the curriculum from the academic year 2011-12 (Žakelj et al., 2011). However, the pedagogical approach in both groups was fundamentally different, as we have emphasized multiple times. In the EG, problem-based instruction was central in all mathematical topics and across various taxonomic levels. In contrast, the CG instruction leaned more towards mastery of algorithms (behaviorist) or teaching specific formulas and computational skills. Even mathematical problems, mostly solvable with a single approach and of an arithmetic nature, were tackled traditionally (gestalt-style), employing a calculation model: problem, answer. Instructions for solving these problems at different presentation levels and guiding an analytical-synthetic process were absent.

Results indicate that despite a minor advantage of the EG in the pre-test, the post-test results revealed that, when accounting for pre-test achievements, EG students outperformed their counterparts in the CG. These findings align with existing literature (Roh, 2003), which we expanded upon by investigating the impact of problem-based learning on students' grasp of new mathematical concepts.

The results of the pre-test indicated a slight superiority of the experimental group (EG) over the control group (CG) in two of the three assessed topics, namely arithmetic (A) and logic (L); no differences were detected concerning geometry (G). When these results were utilized as a control variable in the post-test, the analysis of covariance revealed that the EG outperformed the CG in all three mathematics domains. Therefore, the problem-based approach demonstrated positive effects on all three topics (cf. Putri et al., 2019). This observation may be attributed to the essential role of problem-solving skills in all mathematics domains and the need for a deep understanding of various mathematical topics.

To comprehensively elucidate the reasons behind the superiority of the problem-based learning model compared to the traditional approach, additional research is warranted. Nevertheless, it is acknowledged that problem-based teaching bolsters content acquisition, processes understanding, and metacognition (Downing et al., 2011). This study extends these findings to include conceptual knowledge. Furthermore, literature emphasizes that problem-based teaching integrates real-life scenarios, either teacher-defined or collaboratively created with students (Hung et al., 2008). Students construct new knowledge through exploration, justification, verification, and active participation (Strmčnik, 2001; Van Gog et al., 2020), suggesting that EG students were likely more engaged, gaining a more profound understanding of mathematical concepts by solving real-world problems (cf. Hardy III et al., 2017).

Moreover, future research could delve into students' motivation throughout the experiment. Existing studies reveal that the problem-oriented teaching approach in mathematics requires not only high motivation but also must be supplemented with it to catalyze the learning process (Condell et al., 2010; Dunbar, 2017).

Understanding concepts and problems often proves to be more demanding than procedural knowledge or problem solving. Students frequently execute algorithms or solve problems without grasping the mathematical understanding behind them. We have presented the described concept formation model (arrangement) in separate steps for clarity. However, it's apparent from the content that these "steps" do intermingle, and drawing clear distinctions among them is difficult. When introducing new concepts in the future, it would not require to rigidly separate these steps, but instead depict the development of individual concepts as a comprehensive process. Additionally, at the elementary level, students do not necessarily go through all nine steps (cf. Dörfler, 1991); most commonly, they reach the fifth to seventh step, contingent on the nature and complexity of the concept and their abilities. Thus, teaching should encompass differentiation and individualization.

It is crucial to emphasize that at the elementary level, during the stage of concrete operations, we must never omit the concrete level when forming mathematical concepts, nor should this phase be too brief. The lack of understanding basic mathematical concepts often results from skipping or abbreviating the concrete level. Transitioning from the concrete level to the abstract level in concept acquisition is not the aim of just one class or one day; it is a long-term goal. Some features of this sequence of levels are worth mentioning (Labinowicz, 1989):

- Students can engage with materials even at the highest level.
- Initially, students focus on processes and intuitive relationships before addressing solutions or symbolic representations using mathematical expressions.
- Only after substantial experience on a certain level can students engage in similar activities at a higher presentation level.
- In the classroom, different students can play the same game on different levels at any time. Teachers guide students when transitioning to higher levels.
- Students have the opportunity to initiate the creation of their own problems and presented operations.

While the current research underscores the effectiveness of the problem-based teaching model in enhancing the comprehension of mathematical concepts, it is important to acknowledge certain limitations. Firstly, the sample size is constrained, potentially impacting the generalizability of our findings. Moreover, the absence of randomization might introduce some degree of uncertainty into the results. Additionally, the present study focused solely on grade 3 pupils, therefore different results might be expected among younger or older students. Secondly, the study's geographical confinement to Slovenia could also influence the applicability of the findings to broader contexts. Thirdly, the study's duration solely captured short-term effects on the comprehension of mathematical concepts, leaving unanswered the question of whether this conceptual understanding endures over time. Long-term outcomes remain unaddressed by this present study. Future investigations could expand upon our research and endeavor to mitigate the aforementioned limitations. Despite these constraints, the study underscores the efficacy of employing the problem-based teaching model not only for cultivating procedural and problem-solving skills but also for fostering conceptual understanding.

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