



## **Cognition Processes of ASD Students: Recommendations for Mathematics Teaching and Learning Process**

**Nur Fauziyah**

Dr., Universitas Muhammadiyah Gresik, Indonesia, [nurfauziyah@umg.ac.id](mailto:nurfauziyah@umg.ac.id)

**I Ketut Budayasa**

Prof., Universitas Negeri Surabaya, Indonesia, [ketutbudayasa@unesa.ac.id](mailto:ketutbudayasa@unesa.ac.id)

**Dwi Juniati**

Prof. Dr., Universitas Negeri Surabaya, Indonesia, [dwijuniati@unesa.ac.id](mailto:dwijuniati@unesa.ac.id)

This study aims to describe the cognitive process of autism spectrum disorder (ASD) students in solving mathematical problems in order to provide recommendations towards ensuring an appropriate learning process. It was conducted qualitatively using three ASD students in senior high school in Indonesia with high, moderate, and low intelligence. The student with high intelligence was able to receive and recall all the information in 4 forms, determine patterns, recall relevant information, make generalizations from patterns using two approaches in order to discover general formulas, and compare the general formula with other formulas. Meanwhile, the moderate intelligence student was able to receive and recall all information in two forms, determine patterns using image representation and concrete media, recall relevant information, and understand the pattern but unable to make generalizations towards obtaining a general formula due to the inability to understand the meaning of symbols. Moreover, low intelligence student was able to receive and recall a small piece of information in one form of representation, unable to determine the pattern because the subject was only able to understand qualitative information and unable to discover general formulas, and experienced difficulty in recalling relevant information.

**Keywords:** cognition process, autism spectrum disorder, mathematical teaching, learning process, learning

### **INTRODUCTION**

Cognition is a mental activity in the human brain which starts from the process of receiving information by the five senses or sensory registers followed by filtration, transmission, processing, and storage as well as the recovery of the information in the brain (Solso, 2008). The filtering process is usually conducted due to the fact that information received by the five senses is very large and there is a need for proper and effective selection of those needed while the selection process requires focusing on the

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information through the concept known as “attention”. It is also important to note that some of the filtered information is usually forwarded to the brain for processing through perception or provision of meaning or interpretation based on previous knowledge, thereby, leading to different perceptions depending on the knowledge possessed by each individual (Solso, 2008).

The next stage is internal representation which is the process of constructing information to be stored in memory and made available to be recalled at any time. It is, however, important to note that the cognitive process is usually higher when the information is in the form of a problem due to the use of the results in the decision-making process (Solso, 2008).

Mathematics is a subject with different problems and its competence is mainly determined based on the ability to solve these problems (Shultz, 2020). This means it is focused on problem-solving ability which is an important skill in everyday life. Previous studies showed that good problems allow students to strengthen and expand their knowledge towards stimulating new learning process. However, mathematical problems involving story is usually challenging for many students, especially those with cognitive difficulties, due to the fact that they do not only require math skills but also the ability to read comprehension, reason, and change words and numbers into operation accordingly (Neef et al., 2003).

Another situation is when someone with a social interaction and communication disorder such as an ASD is required to solve a math problem which was discovered to be rarely studied despite the serious increase in the population of ASD students (Bae, 2013). Meanwhile, the term autism was first introduced as "an escape from reality" by Eugen Bleuler, a psychiatrist and psychologist from Switzerland, in 1912 and reported to originate from the Greek word "*autos*" which means 'self' (Frith, 2003). In the mid-20th century, doctors, psychiatrists, and psychologists including Leo Kanner started making attempts to define the group of children with developmental disorders or abnormal development.

Leo Kanner (J. Harris, 2018) was, however, the first to formally publish an article on autism by showing that ASD children are born with a congenital inability to make eye contact. The concept was further defined as an abnormality in the development of social communication indicated by repetitive behavior and limited imagination. These children are usually characterized by language delays, communication difficulties, impaired social interaction, typical behavioral patterns, and huge concern on certain matters of interest. Furthermore, it is possible to suppress the level of autism in children showing severe cases using the appropriate therapy (Chiu et al., 2013).

Hermelin and O'Connor in *Psychological Experiments with Autistic Children* examined three groups of children which have the same mental age, and these include those with autism, ordinary children, and the mentally disabled. The results showed the autistic children were completely unable to perform some of the tasks the other groups were able to perform (Tzavaras, 1972). A preliminary study on 21 ASD children at the

elementary school level within an age range of 8 to 16 years consisting of 15 boys and 6 girls also that they have very different mathematical abilities. Moreover, studies have also been conducted to determine their ability to complete basic mathematical operations, especially addition and subtraction, and the findings showed several differences in their capabilities both in ordinary operations and those involving open-ended and story questions. It was discovered that 15 students were able to add single-digit numbers to single-digit numbers, 13 added two-digit to single-digit, and 10 added two-digit to two-digit numbers. Moreover, the students' knowledge concerning the concept of addition was reevaluated through additional questions in an open-ended form which were accompanied by pictures to match the appropriate numbers in the problem. The result showed only 6 students were able to solve the problem and the ASD students were found to have difficulty solving the open-ended questions even though the operations only involved one-digit numbers (Fauziyah et al., 2021).

The disruption of social interaction, communication, and repetitive behavior of ASD children makes people view them as being retarded children while some classify them as people with mental disorders. They also have learning difficulties due to their inability to adapt to their environment, therefore, they receive less attention in class and are also considered to be interfering with the learning process when grouped with normal children in a class.

The number of ASD children has been reported to be increasing annually (Özerk & Cardinal, 2020; zerk, 2016) with the prevalence in China recorded to be 1% (Sun et al., 2019), approximately 3.4 per 1000 with a 4:1 male-female ratio in the USA (Yeargin-Allsopp et al., 2003), 18.5% in Vietnam (Xuan Tuan et al., 2019), and 6.50 ASD per 1000 children in ASEAN countries (Chiarotti & Venerosi, 2020). This fairly high prevalence requires more attention with more focus on studies to show the cognitive processes of these children in solving mathematical problems in comparison with normal children. This is necessary due to the possible influence of the difference in their intelligence on the cognition process.

The ability to solve mathematical problems is associated with the learning process which is aimed towards sharpening the reasoning and developing the logical and systematic thinking skills of the students. This process usually requires interaction between the teachers and students as well as among the students in the classroom. It has, however, been discovered that current studies on the cognitive processes of students in solving math problems are focused on normal children.

Cognitive ability is closely related to intelligence even though they are not the same. This is due to the fact that human intelligence is defined as the ability to acquire, recall, and use knowledge to understand abstract concepts as well as the appropriate application of concrete relationships between objects and ideas (Solso, 2008). It is, however, important to note that several environmental factors such as education, premature birth, nutrition, pollution, drug and alcohol abuse, mental illness, and developmental disorders affect a person's IQ even though genetics plays an important role. Moreover, there is a close relationship between cognitive abilities and intelligence (Protzko, 2017).

Based on observations in several inclusive schools in Gresik Regency and Surabaya City, Indonesia, it shows there are differences in the level of intelligence for ASD students as well as their cognitive development. Teachers also pay less attention to the differences in the intelligence of the students by considering those with ASD to be the same. Therefore, research is needed to explore in depth the cognitive processes of ASD children in solving math problems based on different levels of intelligence. Thus, the research question is "How is the cognitive process of ASD students in solving mathematical problems based on differences in intelligence levels?". The results are expected to be an input for learning strategies in inclusive classes including ASD children with different levels of intelligence. In addition, the results of this study are expected to provide recommendations in providing different interventions for ASD students with different levels of intelligence.

### **Literature Review**

#### **Cognition Processes in Solving Mathematical Problems**

The cognitive process is a mental process in the human brain usually used to process information (Solso, 2008). These mental activities include the acquisition, organization, and application of knowledge. Moreover, cognition refers to the mental processes through which external and internal inputs are transformed, reduced, interpreted, stored, retrieved, and used. These mental activities also involve different functions such as perception, attention, memory encoding, retention, recall, decision-making, reasoning, problem-solving, drawing, planning, and executing acts (Anderson, 2015).

The cognition process is closely related to how a person perceives, learns, remembers, and thinks about information (Sternberg & Sternberg, 2012). It also involves the acquisition, storage, transformation, and use of knowledge (Anderson, 2015). There is a correlation between these processes and the mental activity in receiving information as well as schemas formed and stored in the cognitive structure of a person (Skemp, 2020). Meanwhile, a scheme is a mental or cognitive structure of an individual usually used to adapt and coordinate the surroundings. This means the knowing process normally starts with the fact that individuals are born with reflexes which allow them to interact with their environment to ensure adaptation (Lutz & Huitt, 2003). Furthermore, this adaptation process is also based on the belief that knowledge building is an activity continuously constructed by individuals. This means knowledge is created and manipulated into cognitive structures through interaction with the environment (Lutz & Huitt, 2003).

A person's cognitive process has also been explained to be working through an information processing theory when there is incoming information (Atkinson & Shiffrin, 1968). This process is further divided into three components which include sensory memory, short-term memory, and long-term memory. The information first enters the sensory register where it resides for a very short period, decays, and gets lost. Moreover, some of the information moves into the short-term memory and later enters the long-term memory. This type is not only stored to be left alone but is retained, manipulated, and changed as the newly acquired knowledge (Lutz & Huitt, 2003).

Working memory refers to the mental workspace which is required to control, organize, and actively maintain relevant information to complete complex cognitive tasks including mathematics (Raghubar et al., 2010). Several factors have been reported to be influencing how memory works on mathematics and these include age, level of mathematical ability, language of instruction, and mathematical tasks characteristics. This, therefore, led to the definition of cognitive process in this study as a mental activity in information processing which involves the acquisition, storage, recall, and organization of information. Further explanation in relation to the process of solving mathematics problems also showed the concept as a mental activity required to receive, store, recall, and process information in an effort to solve mathematical problems. Moreover, the steps applied in this study to solve problems are in line with the Polya's theory (Polya, 2004) and these include (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) evaluating or checking.

### **Mathematical Learning of students with ASD**

There are variations in the abilities and barriers of ASD children, but they have all been discovered to be less interactive with teachers and other students in the classroom, tend to be alone without any desire to interact with their peers in class, show repetitive behavior, and are not willing to make eye contact (Santos et al., 2017). Moreover, those with severe conditions often display a bad temper in class when they experience uncomfortable situations. Therefore, they are most time advised to be placed in a special class with special teachers and a structured learning system. However, some of these ASD children have the ability to study in regular classes to ensure they have more adequate communication as well as cognitive and self-help skills.

The disruptions experienced by ASD children usually hinder their classroom learning process as indicated by the difficulties they have in coping with abstract learning materials such as those associated with mathematics. This means there is a need to apply both concrete and virtual media as another forms of interaction for them to learn mathematics. These methods can assist them in solving mathematical problems accurately and independently, but virtual manipulative media have been reported to be faster than the concrete type. Moreover, the two systems allow the students to solve subtraction problems correctly and also allow them generalize their subtraction knowledge to more real-world applications (Bouck et al., 2014). There are, however, other manipulative media which can also be used to resolve the difficulties in mathematical reasoning for ASD children (Santos et al., 2017).

The appropriate learning method is usually designed in line with the age and learning style of the child (Hao & Layton, 2018) as well as the abilities and barriers experienced during the learning process. This means a combination of several learning methods is normally required to teach these special children. Some of them, however, cannot respond effectively to visual stimuli and this usually leads to the prioritization of learning methods with a lot of visual stimuli (Shin & Bryant, 2017). Moreover, the methods which involve using teaching aids accompanied by certain interventions are another option (Gevarter et al., 2016).

Several studies have been conducted to help provide interventions for ASD children, but none has focused on showing the learning difficulties they experienced based on their cognitive processes when working on math problems. Previous research tends to focus on providing treatment and evaluating the results based on certain abilities being studied (Bae, 2013).

### **The Correlation between ASD Students' Cognitive Processes in Solving Mathematical Problems with their Intelligence Levels**

ASD is a type of autism which is different from and more common than the other types. Some of its attributes include the inability of newborn babies to make eye contact with people, lack of reaction to stimuli, aloofness, and lack of reciprocated social and emotional relationships (“American Psychiatric Association Diagnostic and Statistical Manual of Mental Disorders (DSM-IV),” 2012). This disorder exists from mild, moderate to severe as in other types and those with the severe cases usually suffer from serious barriers to communication and imagination, dislike loud noises, insensitive to pain or fear, persist in routine activities, behave repetitively and stereotypically, and throw tantrums or cry for no apparent reason (Hodges et al., 2020). ASD is usually called true or childhood autism due to the fact that it is experienced by children right from infancy. ASD is normally found in children with low to high intelligence and this is contrary to Asperger's syndrome which generally exists among those with average or even above the average intelligence (Chiang & Lin, 2007).

Previous studies showed that the cognitive development of ASD children was uneven. A study on 120 autistic children consisting of 108 boys and 12 girls which were 73 preschool age and 47 school-age found that they have better nonverbal ability than verbal ability (Joseph et al., 2002). Another study also showed that their cognitive abilities affect the age of diagnosis and the severity of the autistic symptoms such that children with higher IQs are more likely to be identified at a later age and those with IQ below normal show more severe symptoms, including more social problems (S. L. Harris et al., 1991). Moreover, (Long et al., 2011a) also found differences in language skills for more retarded ASD children when compared to the normal ones and a slight difference in their cognitive abilities. The experiment was conducted on a sample of 147 ASD children aged 16-38 months referred to a diagnostic clinic for a developmental evaluation.

The types of cognitive processes that a person uses include remembering, understanding, applying, analyzing, evaluating, or creating. For example, students are often expected to remember facts, understand concepts/principles and apply procedures including applying facts and concepts. In each of the relatively more complex cognitive processes including analyzing, evaluating, and creating, various types of knowledge content are generally used (Haolader et al., 2015). Moreover, cognitive ability has a close relationship with intelligence even though the two are not the same as indicated by some abilities usually used to represent human intelligence such as the ability to (1) classify patterns, (2) modify behavior adaptively, (3) think deductively, (4) think inductively (generalization), (5) develop and use conceptual models, and (6) understand (Sternberg, 2019). It has also been reported that cognitive ability is based on the human

brain and mental processes required to implement any task and is more concerned with the mechanism of how people learn, remember, and pay attention rather than the real knowledge acquired (Latham, 2006). Meanwhile, IQ or *Intelligence Quotient* generally describes scores on tests designed to measure a person's cognitive abilities compared to the general population with the focus on the general ability to solve problems and understand concepts. There is a high positive correlation between IQ and success in school and work but there are several other cases where IQ and success do not correlate. Furthermore, it is possible to train or improve a person's cognitive abilities while intelligence is in the form of relatively static test scores which is the same up to adulthood (Latham, 2006).

## METHOD

### Subjects

The research was conducted using ASD students in *Resource Centers (RC)* including high schools and ASD therapy centers which were classified based on their level of intelligence using the IQ data obtained from the teachers. This led to the selection of 3 ASD students having the same gender, school level, and class but the intelligence levels were high, moderate, and low respectively. Moreover, this study examined the cognition process, and this requires selecting the students with a level of cognitive development at the stage of formal operations. The three subjects used were male students found in Class X SMA and identified as having ASD through the *Diagnostic Statistical Manual - Fifth Edition (DSM- V)* developed by the *American Psychiatric Association (APA)* in 2014. The prevalence of the male gender in the sample is supported by the fact that there are more men with ASD than women at a ratio of 4 times and more (Long et al., 2011a).

The types of autism from the three subjects were classified as moderate with the characteristics of the subject being able to communicate and interact. The subject did not show any symptoms of tantrums. Although the characteristics of autism are still very visible, they are still unable to focus, often laugh alone, stare blankly, and repeat certain words or sentences.

#### *Subject 1: ASD with high intelligence*

This subject was a tenth grader, aged 16 years with an IQ score of 115 diagnosed with ASD since the age of 3 years. The subject received inclusive education at the elementary school and junior high school levels and is currently in high school which is a general education program. The information provided by the parents showed the subject has been receiving intensive therapy since the age of three years in a therapy center located outside the school with the focus on emotional control, self-confidence, and motivation in learning. The therapy activities are conducted twice a week for one hour outside of regular school hours.

#### *Subject 2: ASD with moderate intelligence*

The subject was an 18 years old tenth grader with an IQ score of 95. The information provided by the parents showed the subject was diagnosed with ASD at the age of 3

years, the subject attended general education at the elementary and junior high school levels, and presently in an inclusive senior high school. The subject did not continue his education immediately after graduating from junior high school by waiting for 1 year. Moreover, the subject received intensive therapy from the age of five to nine years in a therapy center to control emotions, self-confidence, and motivation in learning. The therapy activities were conducted twice three times a week and each meeting lasted for 1 hour.

*Subject 3: ASD with low intelligence*

The subject was a 16-year-old tenth grader in high school with an IQ score of 80. The subject was diagnosed with ASD when was 3 years old and the data obtained from the school showed the subject does not respond well to instructions and lacks the ability to process information or problems logically through effective stages. This is normal considering the fact that those with IQ below the average usually have problems understanding concepts, especially when dealing with complex tasks.

**Ethics/permissions**

Permissions were obtained from the Minister of Education and Culture and the relevant schools to conduct this research. Moreover, pseudonyms were used to protect the identity of the participants and were allowed to withdraw from the research at any time without fear or prejudice.

**Setting**

The research was conducted in two different high schools, one private and one public, located in different cities of East Java Province in Indonesia. The ASD student with high intelligence attended a private general education high school while those with moderate and low intelligence attended a public inclusive education high school. The private school is in an industrial area of the city while the public school is in the suburbs of the second-largest city in Indonesia. Moreover, a certified educator is designed to be providing support services for students with ASD in both schools.

**Designs**

The research was conducted using a flexible and evolving style in accordance with the field circumstances and adjustments were made to the factors affecting the results. It is important to note that it is impossible to generalize the findings, but inference can only be made on the actual field phenomenon according to the research objectives. This approach is categorized as qualitative, and the research is said to be descriptive explorative (Solso, 2008). The data used were in the form of words derived from observation, interviews, or documents, and these qualitative data have been discovered to be richer in terms of descriptions and explanations when compared with quantitative data (Miles et al., 2014).

### **Instruments and their validation**

The primary instrument was the researcher while the secondary instruments were the math problem-solving test and interview guidelines provided. The test was in the form of a worksheet containing mathematical problems arranged based on the curriculum used at the high school level. The content of each problem was validated based on *content validity* to ensure the construction, material, and language used for the questions are effective and valid. This test was further used to collect data on the cognitive processes used by the subjects in solving mathematical problems according to Polya's problem-solving procedure.

The criteria used to validate the questions' construction include (1) sentence not providing multiple interpretations, (2) information provided is sufficient and not excessive to solve the problem, and (3) the question is formulated in the form of an interrogative sentence. Moreover, the instrument also passed through an expert validation process which consists of material-based and language-used-based validation before it was applied to collect the data needed for this research.

Researchers used triangulation techniques to ensure the validity of the data. The type of triangulation used by researchers is time triangulation. In this triangulation, the researcher arranges a second mathematical problem test instrument which is equivalent to the first mathematical problem test instrument. The second mathematical problem test instrument was tested on the research subjects a few days after the subject completed the first mathematical problem test instrument.

### **Data collection and analysis procedures**

Data were collected using task-based interviews through a think-aloud method. The subjects were made to work on their math problems by writing answers on the sheet provided as well as through the expression of their thoughts. Moreover, open-ended questions were asked when their thoughts were not expressed in order to obtain responses. All the activities of the subjects during the process of solving the math problems were recorded using a video recorder. Furthermore, the think-aloud method was applied effectively to obtain qualitative data and the method has been discovered to have a strong theoretical basis to provide valid data on the thoughts of a subject (Charters, 2003).

To analyze the data from task-based interviews, the stages were carried out, namely data categorization, data reduction, data display and drawing conclusions. Categorization of data in this study is based on the Polya stage, then the data is re-categorized based on the type of cognitive process revealed. The data reduction stage is carried out by the selection process, focusing on simplification, and data transformation in the field. Data reduction in this study is an activity that refers to the process of selecting data related to the indicators of students' cognitive processes in solving mathematical problems. Furthermore, the data that has been selected and identified is coded to determine the origin of the data source. The next stage is data display, which is writing an organized and categorized data set so that it is easier to interpret the data to draw conclusions from the data.

## FINDINGS

### Subject 1: ASD Student with High Intelligence

#### *The stage of understanding the problem*

The subject was able to receive all information, stored them in the short-term memory, and recalled them in 4 different forms of representation which include 1 image, 2 verbal, and 1 symbol as indicated in Figure 1.

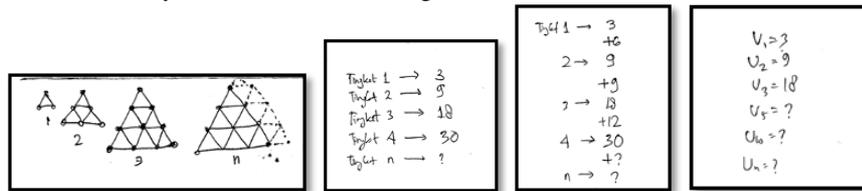


Figure 1

The visual representation used by high-intelligence ASD subjects to retrieve information

The subject recalled the information stored and processed them to determine those considered important, what is known, what is being asked, adequacy, and also understood the patterns very well. The information stored in long-term memory on the concepts directly related to the newly received information was also recalled.

#### *The stage of devising a plan*

The subject was able to recall and represent the information received from the problem in the form of images or appropriate concrete media as the first step to prepare a problem-solving plan. The images and concrete media were later processed to understand the pattern as indicated in the addition of triangles from level 3 to level 4 in Figure 3. The process involved adding the triangle at the end of each 3rd level triangle (a, b, c, and d) as an effort towards developing a settlement plan to determine the general formula.

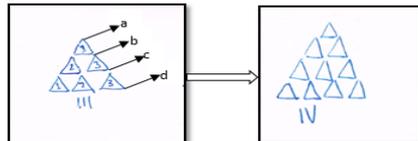


Figure 2

The addition of triangles from the 3rd to the 4th level by the subject

The subject was discovered to have the ability to develop a strategy to solve the problem well at this stage through two different approaches using the image and concrete media. Generalizations were also made from a pattern to develop a complete plan to discover the general formula.

#### *The stage of implementing the plan.*

The subject recalled and represent the information received from the problem in the form of images and concrete media which were further processed as an effort to understand the pattern and create the generalization needed to determine the general

formula. This means the two approaches used to obtain the same two general formulas include (1) understanding and generalizing the pattern involved in adding triangles from one level to the next and (2) using a number sequence showing the number of matchsticks in each level after which the different patterns of the adjacent term were generalized as indicated in Figure 3.

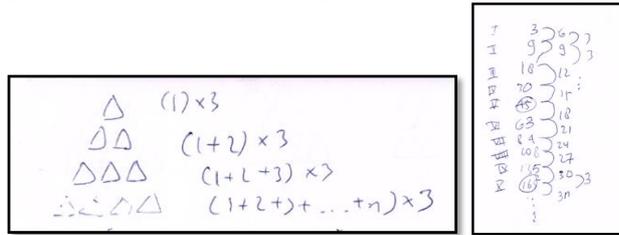


Figure 3

The generalization pattern of adding triangles to the nth level and generalizing different patterns of the closest term

This stage shows the subject was able to remember the completion plan prepared and the steps were used in the implementation process with little improvement to make sure the pattern obtained was correct.

*The stage of evaluation*

The subject recalled the information received from the problem in order to check the image pattern produced and the general formula found was evaluated to be true using other formulas. The important parts in the completion step, especially the pattern found, were evaluated as indicated in Figure 4.

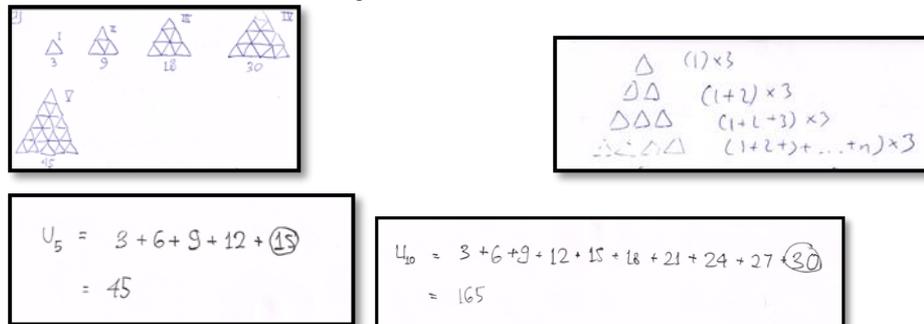


Figure 4

Key points checked by the subject

The subject was also able to use other formulas previously learned to evaluate the truth of the general formula. The overall cognitive process of the ASD subject with high intelligence in solving problems is, therefore, described as shown in Figure 6.

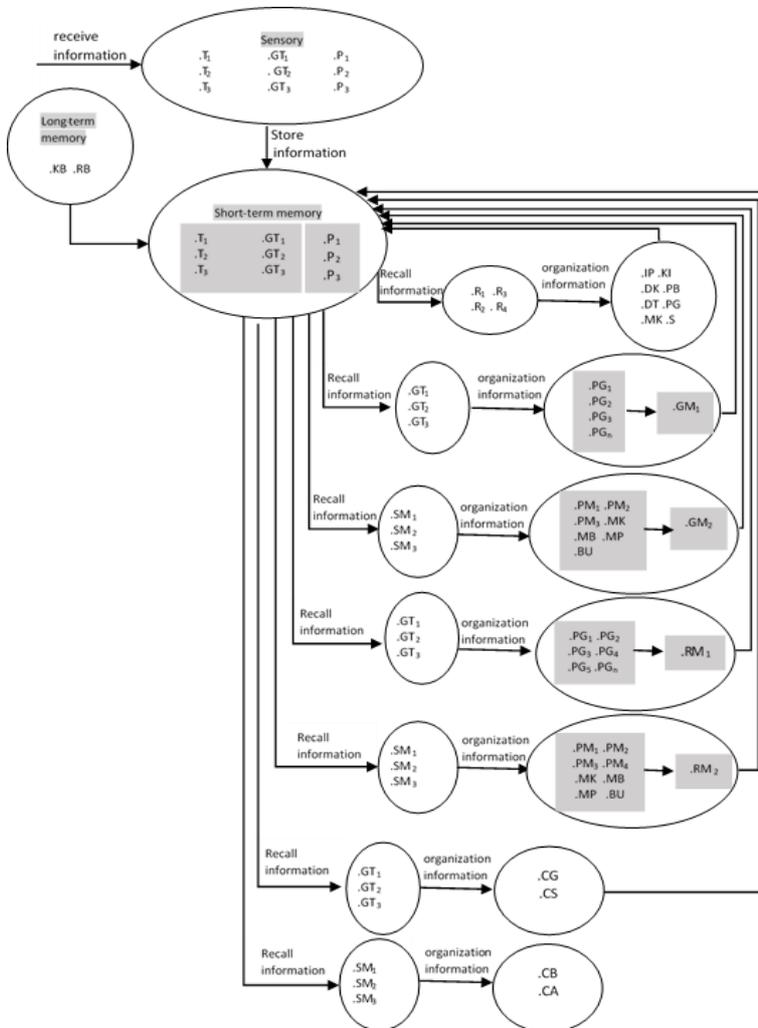


Figure 5  
The cognitive process of ASD subject with high intelligence in solving math problems

- T<sub>1</sub> : Information about the 3 matchsticks needed to create the 1<sup>st</sup> level triangle
- T<sub>2</sub> : Information about the 9 matchsticks needed to create the 2<sup>nd</sup> level triangle
- T<sub>3</sub> : Information about the 18 matchsticks needed to create the 3<sup>rd</sup> level triangle
- GT<sub>1</sub> : The image of the 1<sup>st</sup> level triangle
- GT<sub>2</sub> : The image of the 2<sup>nd</sup> level triangle
- GT<sub>3</sub> : The image of the 3<sup>rd</sup> level triangle
- P<sub>1</sub> : The question of the number of matchsticks needed to form the 5<sup>th</sup> level triangle?
- P<sub>2</sub> : The question of the number of matchsticks needed to form the 10<sup>th</sup> level triangle?
- P<sub>3</sub> : The question of the number of matchsticks needed to form the n<sup>th</sup> level triangle?
- KB : Sequence concept
- RB : Arithmetic sequence formula level 2
- R<sub>1</sub> : Visual representation in the form of images

- R<sub>2</sub> : Visual representation by listing the number of matchsticks starting at levels 1, 2, 3, 4 and n
- R<sub>3</sub> : Visual representation by listing the number of matchsticks starting at levels 1, 2, 3, 4, and n and adding the difference between two adjacent terms
- R<sub>4</sub> : Visual representation in the form of terms symbols
- IP : Determine the important information in the question
- DK : Understanding what is known
- DT : Understand what's being asked
- PB : Understanding patterns of the sequence
- PG : Understanding the 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> level of triangle patterns
- KI : Understanding the adequacy of the information
- MK : Reducing the known
- S : Understanding symbols in problems
- PG<sub>1</sub> : Understanding how to draw the 2<sup>nd</sup> level triangle by adding an image of 2 triangles to the first level triangle
- PG<sub>2</sub> : Understanding how to draw the 3<sup>rd</sup> level triangle by adding 3 triangles to the 2<sup>nd</sup> level triangle
- PG<sub>3</sub> : Understanding how to draw a 4<sup>th</sup> level triangle by adding 4 triangles to the 3<sup>rd</sup> level triangle
- PG<sub>n</sub> : Finding the pattern of adding triangles from one level to the next level to make generalization towards obtaining the *n*<sup>th</sup> level triangle by adding *n* triangles on the *n*-1 level
- GM<sub>1</sub> : Planning on using the pattern of adding triangles from one level to the next to find the *n*<sup>th</sup> term formula
- SM<sub>1</sub> : Level 1 triangular arrangement using media.
- SM<sub>2</sub> : Level 2 triangular arrangement using media.
- SM<sub>3</sub> : Level 3 triangular arrangement using media.
- PM<sub>1</sub> : Understanding how to arrange matchsticks on the 2<sup>nd</sup> level triangle by adding 2 triangles to the first level triangle arrangement.
- PM<sub>2</sub> : Understand how to arrange matchsticks on the 3<sup>rd</sup> level triangle by adding 3 triangles to the 2<sup>nd</sup> level triangle arrangement
- PM<sub>3</sub> : Understand how to arrange matchsticks on the 4<sup>th</sup> level triangle by adding 4 triangles to the 3<sup>rd</sup> level triangle arrangement
- MK : Counting matchsticks in a triangular arrangement of levels 1, 2 and 3
- MB : Writing down the number that shows the amounts of matchsticks in the 1st, 2nd, and 3rd level triangles in a row
- MP : Determining the difference between adjacent term
- BU : Finding out the patterns of differences between adjacent terms and generalizing them to determine the general form of differences between adjacent terms, namely  $3n$
- GM<sub>2</sub> : Plotting through the pattern of differences between adjacent terms will be used to find the formula for the *n*<sup>th</sup> term
- RM<sub>1</sub> : Using the pattern of adding triangles from one level to the next and linking them to the concept of triangles to find the formula for the *n*<sup>th</sup> term, i.e.  $U_n = (1 + 2 + 3 + \dots + n)3$
- RM<sub>2</sub> : Using the difference pattern between adjacent terms, the first term and the first difference of the sequence to find the formula for the *n*<sup>th</sup> term, i.e.  $U_n = 3 + 6 + 9 + \dots + 3n$
- CG : Checking the level 1, 2, 3, 4, and 5 triangle arrangement images that have been drawn
- CS : Checking the pattern of adding triangles from one level to the next level
- CB : Checking the appropriateness of using the formula *n*<sup>th</sup> term by circling the last number in the summation to determine the *n*<sup>th</sup> term
- CA : Checking the two formulas of the *n*<sup>th</sup> term found using the arithmetic sequence formula level 2

**Subject 2: ASD students with moderate intelligence**

*The stage of understanding the problem*

The subject was able to receive all the information, stored them in short-term memory, and recalled them in two forms of visual representations which include (a) images accompanied by the symbols of the sequences U<sub>1</sub>, U<sub>2</sub>, and U<sub>3</sub> and (b) listing the number of matchsticks starting at level 1, 2 and 3 by writing the syllable symbols of the sequence from U<sub>1</sub>, U<sub>2</sub>, and U<sub>3</sub>, and adding the difference between two adjacent terms.

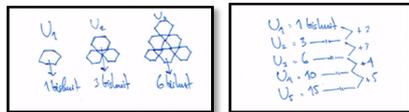


Figure 6  
Two forms of visual representation used by the subject to recall information

The information stored was processed to determine what is known, what is being asked, important information, adequacy of information, and to understand the patterns effectively. The subject also recalled the information on the concepts related to the newly received information from the long-term memory.

#### *The stage of devising a plan*

The subject was able to recall the information in the problem and represented them in the form of concrete and image media as the first step to prepare a completion plan. The concrete and image media were used by the subject to understand the pattern as indicated by the pattern used to add triangles on a certain level from the previous level. However, the subject was unable to generalize the pattern to obtain a general formula as indicated in Figure 7 which shows the triangular levels along with the differences observed.

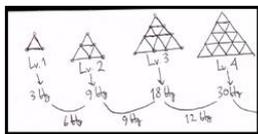


Figure 7

The pattern of triangular levels and differences between levels drawn by ASD subject

The subject was able to develop a complete strategy needed to determine the triangle pattern at levels 4, 5, 6, and others using the image approach and concrete media at this stage. Meanwhile, the pattern was unable to be used to generalized to determine the general formula.

#### *The stage of implementing the plan*

The subject recalled the information from the problem and represented them visually in the form of concrete and image media which were further used to understand the pattern from one triangular level to the next in order to determine the difference from one level to another adjacent level. This allowed the subject to provide a rational reason that the triangular arrangement using media was correct due to the fact that the level with the higher number of arrangements requires more matchsticks than the previous. However, the subject was unable to use the knowledge of the pattern for the generalization and development of a general formula. It was also concluded at this stage that the subject remembered and implemented the completion plan in accordance with the previous preparation.

#### *The stage of evaluation*

The subject recalled the information received from the questions used to check the pattern of the image produced and checked the important parts of the implementation stage as indicated in Figure 8.

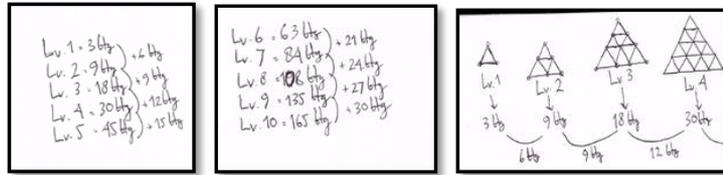


Figure 8  
Important parts checked by the ASD subject

This is an interesting observation as regards the cognitive process of ASD students in solving mathematical problems through the stages of understanding the problem, devising a plan, carrying out the plan, and evaluation. It is possible to compare these findings with the results of previous studies. The general cognitive process of ASD students with moderate intelligence in solving problems is, therefore, presented in Figure 9.

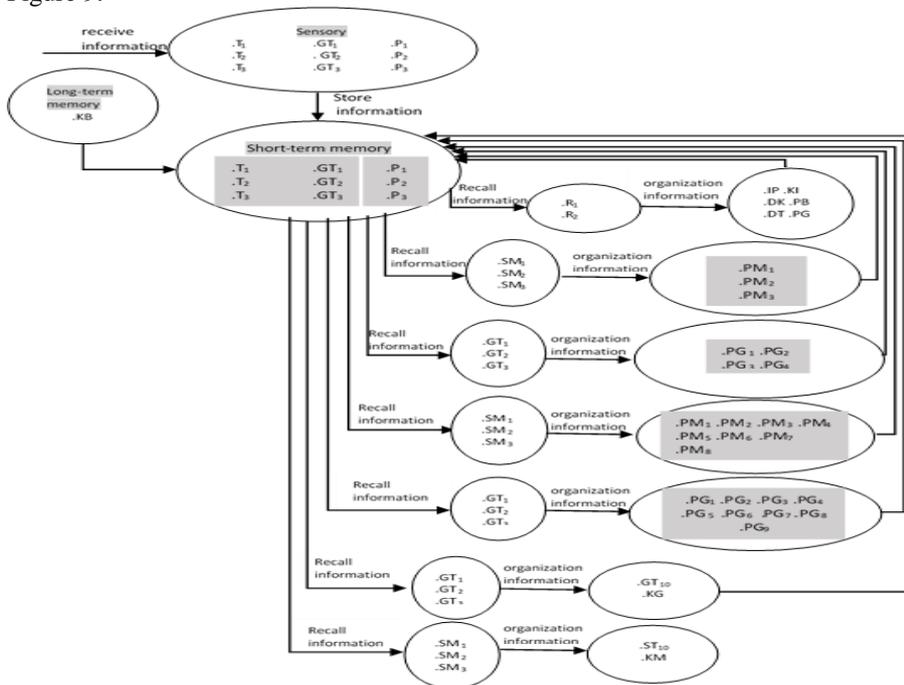


Figure 9  
The cognitive process of ASD subjects with moderate intelligence in solving math problems

- T<sub>1</sub> : Information about the 3 matchsticks needed to create the 1<sup>st</sup> level triangle
- T<sub>2</sub> : Information about the 9 matchsticks needed to create the 2<sup>nd</sup> level triangle
- T<sub>3</sub> : Information about the 18 matchsticks needed to create the 3<sup>rd</sup> level triangle
- GT<sub>1</sub> : The image of the 1<sup>st</sup> level triangle
- GT<sub>2</sub> : The image of the 2<sup>nd</sup> level triangle
- GT<sub>3</sub> : The image of the 3<sup>rd</sup> level triangle

P <sub>1</sub>	:	The question of the number of matchsticks needed to form the 5 <sup>th</sup> level triangle?
P <sub>2</sub>	:	The question of the number of matchsticks needed to form the 10 <sup>th</sup> level triangle?
P <sub>3</sub>	:	The question of the number of matchsticks needed to form the <i>n</i> <sup>th</sup> level triangle?
KB	:	Sequence concept
R <sub>1</sub>	:	Visual representation in the form of pictures accompanied by the symbols term of the sequence U <sub>1</sub> , U <sub>2</sub> and U <sub>3</sub>
R <sub>2</sub>	:	Visual representation by listing the number of matchsticks starting at levels 1, 2, and 3 by writing the symbols term of the sequence starting from U <sub>1</sub> , U <sub>2</sub> and U <sub>3</sub> and adding the difference between two adjacent terms
S	:	Understanding symbols in problems
IP	:	Determining the important information in the question
DK	:	Understanding what is known
DT	:	Understand what's being asked
PB	:	Understanding patterns of the sequence
PG	:	Understanding the 1 <sup>st</sup> , 2 <sup>nd</sup> , and 3 <sup>rd</sup> level of triangle patterns
KI	:	Understanding the adequacy of the information
SM <sub>1</sub>	:	Level 1 triangular arrangement using media
SM <sub>2</sub>	:	Level 2 triangular arrangement using media
SM <sub>3</sub>	:	Level 3 triangular arrangement using media
PM <sub>1</sub>	:	Understanding how to arrange matchsticks on the 2 <sup>nd</sup> level triangle by adding 2 triangles to the first level triangle arrangement.
PM <sub>2</sub>	:	Understand how to arrange matchsticks on the 3 <sup>rd</sup> level triangle by adding 3 triangles to the 2 <sup>nd</sup> level triangle arrangement
PM <sub>3</sub>	:	Understand how to arrange matchsticks on the 4 <sup>th</sup> level triangle by adding 4 triangles to the 3 <sup>rd</sup> level triangle arrangement
MK	:	Counting matchsticks in a triangular arrangement of levels 1, 2 and 3
MB	:	Writing down the number that shows the amounts of matchsticks in the 1st, 2nd, and 3rd level triangles in a row
MP	:	Determining the difference between adjacent term
BS	:	Find out the pattern of the difference between adjacent terms to determine the 5th and 6th terms.
PG <sub>1</sub>	:	Understanding how to draw the 2nd level triangle by adding 2 triangle images to the first level triangle
PG <sub>2</sub>	:	Understanding how to draw the 3 <sup>rd</sup> level triangle by adding 3 triangles to the 2 <sup>nd</sup> level triangle
PG <sub>3</sub>	:	Understanding how to draw the 4 <sup>th</sup> level triangle by adding 4 triangles to the 3 <sup>rd</sup> level triangle
PG <sub>4</sub>	:	Understanding how to draw the 5th level triangle by adding 5 triangles to the 4th level triangle
PM <sub>4</sub>	:	Understanding how to arrange matchsticks on the 5th level triangle by adding 5 triangles to the 4th level triangle arrangement
PM <sub>5</sub>	:	Understanding how to arrange matchsticks on the 6th level triangle by adding 6 triangles to the 5th level triangle arrangement
PM <sub>6</sub>	:	Understanding how to arrange matchsticks on the 7th level triangle by adding 7 triangles to the 6th level triangle arrangement
PM <sub>7</sub>	:	Understanding how to arrange matchsticks on the 8th level triangle by adding 8 triangles to the 7th level triangle arrangement
PM <sub>8</sub>	:	Understanding how to arrange matchsticks on the 9th level triangle by adding 9 triangles to the 8th level triangle arrangement
PM <sub>9</sub>	:	Understanding how to arrange matchsticks on the 10th level triangle by adding 10 triangles to the 9th level triangle arrangement
PG <sub>5</sub>	:	Understanding how to draw the 6th level triangle by adding 6 triangles to the 5th level triangle
PG <sub>6</sub>	:	Understanding how to draw the 7th level triangle by adding 7 triangles to the 6th level triangle
PG <sub>7</sub>	:	Understanding how to draw the 8th level triangle by adding 8 triangles to the 7th level triangle
PG <sub>8</sub>	:	Understanding how to draw the 9th level triangle by adding 9 triangles to the 8th level triangle
PG <sub>9</sub>	:	Understanding how to draw the 10th level triangle by adding 10 triangles to the 9th level triangle
ST <sub>10</sub>	:	10 <sup>th</sup> level of the triangular arrangement
GT <sub>10</sub>	:	10 <sup>th</sup> level triangle image
KG	:	Assuring the truth of the image that has been made
KM	:	Assuring the media arrangement that has been made

### ***Subject 3: ASD students with low intelligence***

#### *The stage of understanding the problem*

The subject was able to receive information and store them in the short-term memory after which they were recalled as a visual representation in the form of an image as indicated in Figure 10.

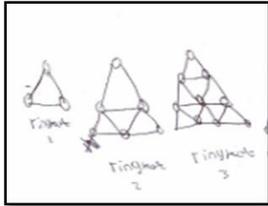


Figure 10

The visual representation used by the ASD subject to call information

The subject was able to recall information in the questions to determine what is known and what is being asked but was unable to determine the important information and adequacy of the information in the question. It was also discovered the subject was unable to recall the related information stored in long-term memory.

#### *The stage of devising a plan*

The subject recalled the information in the question concerning the image of the triangle arrangement at this stage and made a plan to obtain the next triangle pattern by adding another triangle to the previous pattern using matchsticks but was unable to plan the visual representation of the additional pattern either through images or concrete media.

#### *Stages of carrying out the plan*

The subject stored and recalled information on triangle arrangement at levels 1, 2, and 3 appropriately in the form of media representation through matchsticks but did not understand the pattern. This, therefore, led to less precision in the arrangement of the matchsticks to form the 4th level.

#### *The stage of evaluation*

The subject stored and recalled information on the triangle arrangement of levels 1, 2, and 3 in the questions to check the answers but was unable to understand the patterns effectively and this made him believe the answer is right even though it was actually wrong. This is another interesting observation as regards the cognitive process of ASD students in solving mathematical problems through the stages of understanding the problem, devising a plan, carrying out the plan, and evaluation. The general cognitive process of ASD students with low intelligence in solving problems is, therefore, presented in Figure 11.

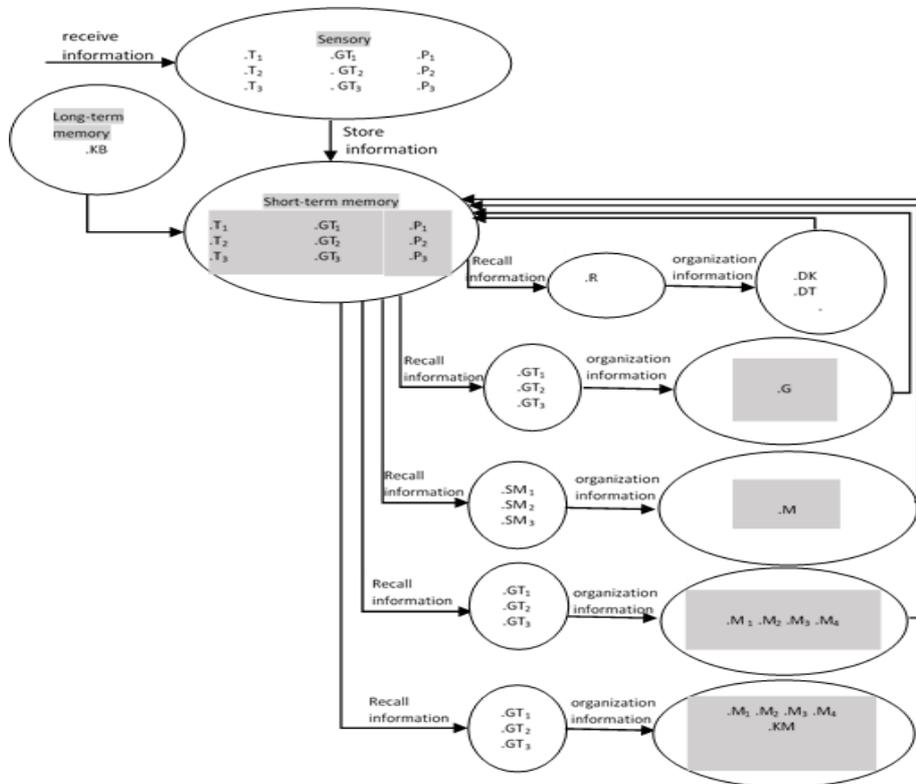


Figure 11

The cognitive process of ASD subject with low intelligence in solving math problems

$T_1$  : Information about the 3 matchsticks needed to create the 1<sup>st</sup> level triangle

$T_2$  : Information about the 9 matchsticks needed to create the 2<sup>nd</sup> level triangle

$T_3$  : Information about the 18 matchsticks needed to create the 3<sup>rd</sup> level triangle

$GT_1$  : The image of the 1<sup>st</sup> level triangle

$GT_2$  : The image of the 2<sup>nd</sup> level triangle

$GT_3$  : The image of the 3<sup>rd</sup> level triangle

$P_1$  : The question of the number of matchsticks needed to form the 5<sup>th</sup> level triangle?

$P_2$  : The question of the number of matchsticks needed to form the 10<sup>th</sup> level triangle?

$P_3$  : The question of the number of matchsticks needed to form the  $n^{\text{th}}$  level triangle?

$R_1$  : Visual representation in the form of images

$DK$  : Understanding what is known

$DT$  : Understanding what's being asked

$G$  : Understanding how to determine the triangle arrangement of the next level by adding the image on the last row of the previous level.

$SM_1$  : Level 1 triangular arrangement using media.

$SM_2$  : Level 2 triangular arrangement using media.

$SM_3$  : Level 3 triangular arrangement using media.

$M$  : Understanding how to determine the triangle arrangement of the next level using matchstick media by adding the arrangement to the last row of the previous level.

$M_1$  : Arranging the 1<sup>st</sup> level of triangle arrangement using matchsticks.

$M_2$  : Arranging the 2<sup>nd</sup> level of triangle arrangement using matchsticks.

$M_3$  : Arranging the 3<sup>rd</sup> level of triangle arrangement using matchsticks.

$M_4$  : Arranging the 4<sup>th</sup> level of triangle arrangement using matchsticks even though it is not quite right.

$KM$  : Assuring in the media arrangement that has been made

## DISCUSSION

### Subject 1: ASD students with high intelligence

The stage of understanding the problem shows the ASD student with high intelligence levels was able to receive, store, recall, and even represent the information in a good four different forms. The observation is, however, not in line with a previous study which used 32 ASD students with high intelligence levels and found that they have difficulties in abstract thinking and information processing (Happé et al., 2006). Moreover, the student had the ability to recall the information stored and processed them to determine those considered important, what is known, what is being asked, adequacy of the information, and also to understand the patterns very well as indicated by the continuous drawing of the triangle pattern in the problem. The subject's ability to represent problems into symbols and images will make it easier to understand mathematical problems. The ability to understand problems is a very important stage to be able to solve mathematical problems well (Misu et al., 2019).

The subject was also able to connect the concepts stored in long-term memory with those in the new problems. The ability to connect between concepts or ideas in mathematics is very important to make it easier to get strategies in solving mathematical problems. The combination of previous mathematical concepts in a coherent manner will be the main basis in understanding further mathematical concept problems (Ma'ulah & Juniati, 2020). The finding was discovered to be different from the result of Williams, Goldstein, and Minshew (Williams et al., 2006) which used 38 ASD students with high intelligence that the memory was relatively less for complex visual and verbal information. Meanwhile, the result is in line with the findings of some previous studies which indicate the ability of these students to express their thoughts in written form even though it is sometimes in the form of short sentences or pieces which require further instructions when deeper cognitive processes need to be explored. They were also found to be in accordance with Long, Gurka, and Blackman (Long et al., 2011b) which compared the cognitive abilities of students with and without ASD and found the language skills of ASD students to be lower than those of non-ASD students.

The subject's ability to solve problems is generally very good and this is in line with the findings of (Glanzman, 2010) that autistic children with average and high intelligence known as *High-functioning Autism* (HFA) have the potentials to increase productivity and develop specific skills. The finding also agrees with the results of Chiang and Lin (Chiang & Lin, 2007) that most autistic students with average and high intelligence or HFA (*High-functioning Autism*) have average while some possess very superior mathematical abilities as well as the findings of Iuculano et al. (Iuculano et al., 2014) that ASD students with high intelligence have high abilities in the field of mathematics.

### Subject 2: ASD students with moderate intelligence

The subject was able to receive all the information, store them in short-term memory, and recall them in two different forms of visual representation at the stage of understanding the problem. This is in accordance with the research by Titeca, Roeyers, Josephy, Ceulemans, & Desoete (Titeca et al., 2014) to compare 33 ASD students having HFA with 54 non-ASD students and found the mathematical ability of some

ASD students to be higher than some of those without the disorder. The process of receiving, storing and recalling this information is an important part of learning, even though ASD students actually have impairments in communication and interaction. This is because the learning process cannot be separated from interaction and communication to receive information, store it in short-term memory and recall it.

The subject was able to recall the information in the problem and present them in the form of concrete media at the stage of devising the plan. The finding was observed to be in correlation with a study by Bouck, Satsangi, Doughty, & Courtney (Bouck et al., 2014) on 3 ASD students aged about 6 to 7 years which showed their ability to solve subtraction problems correctly and generalize the concept through the real manipulative media. Moreover, the results also showed that the subject has the ability to develop a settlement strategy to determine the triangular pattern at levels 4, 5, 6, and others using a detailed image and concrete media approach. This is in line with the findings of Carnahan and Williamson (Carnahan & Williamson, 2013) that ASD students tend to think deeply in completing their assignments as observed from the analysis of 3 ASD students with HFA at the high school level.

Learning media is one of the most important elements in the learning process in the classroom in addition to the learning model. Associated with the characteristics of mathematics, abstract objects, hierarchical concepts and principles, and the processing procedures that manipulate many forms make students often experience difficulties. Visualization of mathematical concepts in the form of concrete objects that can be sensed well by students will help students understand the concept. From this research, the results are very encouraging because ASD students with moderate intelligence are able to visualize abstract mathematical concepts into a concrete form.

The subject was also able to determine the patterns at the problem-solving stage but was unable to generalize the pattern to produce a general formula. The process of generalizing from patterns to general formulas is very important, so an understanding of the meaning of symbols or variables in mathematics is needed. This is the difference observed from subject 1 with a higher IQ and this indicates the possibility of the influence of IQ differences on cognition processes considering subjects having ASD with HFA. The finding is in line with the results of Whitby and Mancil (Whitby & Mancil, 2009a) that reading, writing, and mathematics skills in ASD students with HFA are directly proportional to or commensurate with IQ levels.

### **Subject 3: ASD students with low intelligence**

The subject was discovered not to have the ability to recall the related information stored in long-term memory during the process of understanding the problem. This is in agreement with the findings of Boucher, Bigham, Mayes, & Muskett (Boucher et al., 2008) that ASD students with LFA (*low functioning autism*) have language and memory disorders. So that repetition is needed in understanding mathematical concepts so that the concepts can really be embedded properly. Moreover, the subject was also unable to plan the pattern of the triangles to be added in detail either through images or concrete media. The finding was observed to be the same with the results of Preissler that ASD students with LFA have limitations in determining the simple pattern, have cognitive

difficulties in learning the relationship between images and words, and also found it hard to understand the symbolic nature of images.

The subject did not understand the pattern at the problem-solving stage, thereby, limiting his ability to solve the problem properly (Fauziyah et al., 2020). The result is in accordance with the findings of Mayes, Calhoun, & Murray (Mayes et al., 2011) that students with ASD with HFA had significantly higher cognitive abilities and higher scores on social interaction and behavioral patterns than those with LFA.

These interesting findings related to the cognitive process in solving math problems for ASD students with high, moderate, and low intelligence showed the influence of the intelligence level. This was indicated by the difference in their cognitive processes when solving mathematical problems which is in agreement with the previous findings of Whitby and Mancil (Whitby & Mancil, 2009b) that reading, writing, and mathematics skills in ASD students with HFA are directly proportional or equal to their IQ levels. The result also agrees with the Dickerson and Calhoun's research which showed (Mayes & Calhoun, 2003b) that ASD students are strong in the fields of reading, writing, and mathematics, their abilities matched their IQ levels, and are weak in the areas of attention and social interaction.

A subsequent study by Dickerson and Calhoun (Mayes & Calhoun, 2003a) also showed the academic profile of ASD students in different subjects, and their mathematics and spelling abilities were found to be positively correlated with their IQ. Mayes & Calhoun (Dickerson Mayes & Calhoun, 2003) also researched ASD students with high IQ and showed that they have high scores in reading and mathematics.

#### **CONCLUSION AND SUGGESTION**

The findings showed that the difference in the intelligence of the three subjects affected their cognitive processes in solving mathematical problems. This means there is a need to accommodate differences in cognitive processes of ASD students based on their IQ while teaching mathematics. Different interventions can be introduced through the provision of assignments such as face-to-face and online independent learning with teaching materials developed in the form of complete modules which include understanding, repetition, and enrichment aspects.

The ASD student with high intelligence cognition was discovered to have a very complex process at every stage involved in solving math problems which can exceed those associated with non-ASD students (Fauziyah, Budayasa, et al., 2019) (Fauziyah, Lant, et al., 2019). This means both ASD students with high IQ and those without the disorder can learn together in superior classes of inclusive schools. The subject was able to recall information stored in long-term memory that is directly related to the problem to be solved. The subject was able to make generalizations from a pattern in order to develop a plan of completion that leads to the discovery of a general formula. The pattern is used to generalize to a general formula with two different approaches. Thus, ASD students with high intelligence have reached a high level of thinking, so they need more enrichment training, problems that require analytical skills and critical thinking as challenges.

The ASD student with moderate intelligence was able to receive all information, store it in short-term memory and recall it in two forms of visual representation. The information was processed to determine what is known and what is asked, important information, adequacy of information and understanding patterns well. The subject also recalled information stored in long-term memory that was directly related to the newly received information. Representations in the form of concrete media and images were used as the first step in developing a settlement plan. The representation was used by the subject to understand the pattern, but from the pattern understood, the subject was unable to make generalizations towards obtaining a general formula. This is because the subject does not understand the meaning of mathematical symbols or variables in the problem. Thus, ASD students with moderate intelligence requires more training to convert or symbolize everyday problems into mathematical symbols through mathematical communication.

The ASD student with low intelligence was unable to determine the important information in the problem and was unable to determine the adequacy of the information in the problem. Subject was also unable to recall related information stored in long-term memory. Inappropriate representation of images and concrete media proves that the subject was not able to understand the pattern well. The ASD student with low intelligence was discovered not to have the ability to understand the patterns due to the fact that the subject only thinks based on qualitative cognitive processes without the quantitative aspect. The ASD students with low intelligence require more assistance in repeating practice questions or drills followed by mathematical symbolism. Giving detailed mathematical assignments on understanding basic operating concepts is also needed, so that they are not only able to think qualitatively but also quantitatively when they observe mathematical objects. Moreover, the task deadline in the independent study assignment was designed to be face-to-face and it involved a discussion to report learning performance in order to complete the required study.

Furthermore, further studies are recommended to focus on students with other types of needs with respect to the differences in the subject's attributes such as gender, learning style, level of mathematical ability, and others.

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