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The Effect of Metacognitive Teaching and Mathematical Prior Knowledge on Mathematical Logical Thinking Ability and Self-Regulated Learning

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This paper reports the findings of a quasi-experiment that aims to investigate the role of metacognitive teaching-learning (MTL) and mathematical prior knowledge (MPK) on students' mathematical logical thinking ability (MLTA) and self-regulated learning (SRL). The subject of the study was 70 tenth-grade students inSumedang, Indonesia. One experimental group and one control group were used, and the students were grouped into three categories based on their mathematical prior knowledge (high, medium and low). The instruments of this study were a short-answer-MPK test, an essay MLTA test, and a SRL scale. The data were analyzed by using Anova, *t*-test, Mann-Whitney test, and Kruskal-Wallis test. The study find that compared to conventional learning, MTL gave better influence on students with medium MPK in MLTA and gave an overall better effect on students in SRL. However, for entirely students, the students' grades of SRL are fairly good, but the students' grades of MLTA are still low. There is no interaction between learning approaches and MPK on improving MLTA, and there is an interaction between learning approaches and MPK on improving SRL.

Keywords: metacognitive teaching-learning, mathematical logical thinking ability, self-regulated learning, mathematical prior knowledge, teaching, learning

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INTRODUCTION

Basically, mathematical logical thinking ability (MLTA) and self-regulated learning (SRL) are two of important mathematics learning outcomes that should be improved on Senior High School students. The importance of possessing MLTA and SRL by the students has been in line with the goals of mathematics teaching (NCTM, 2000; Permendiknas, 2006; Kemendikbud, 2012). The goals among others are: a) to draw reason based on mathematical patterns and traits, to draw generalization, to draw conclusion by using rules of inference, to prove, and to clarify mathematical statements; b) to improve self-confidence, to appreciate the beauty of orderliness of mathematics, to perform objective and opened attitude, to be curious, to demonstrate interest and attention on mathematics learning. The first goal illustrated some traits of MLTA, and the second one illustrated part of SRL.

Some studies found that MLTA was still difficult tasks for many senior high students. Even that, difficulty not only for senior high school students but for tertiary students as well (Wason,as cited in Markovits&Barrouillet,2004). In many cases, an adult cannot logically give an exact response to the reasoning cases. Though some researchers state that youngest kids were able to give logical reasoning through abstract or fake premises, but this kind of reasoning was hard for some adults and even for educated person as well (George, as cite in Markovits&Barrouillet, 2004). Some studies (Sumarmo, 1987; Sumarmo et al., 2012; Rohaeti et al., 2014; Setiawati, 2014) reported similar findings namely: students' MLTA were at low to medium grades. To overcome students' difficulties on MLTA, it is necessary to design an innovative teaching-learning approach which gives opportunity to students to exercise to think logically. One of that predicted innovative teaching approach is metacognitive teaching-learning approach (MTLA), which has three main steps namely: modeling, metacognitive scaffolding, and pairs discussion, group discussion, and class discussion. In MTLA, students are awaken for controlling and assessing their thinking process through posing questions and then they attempt to answer accompany with relevant reasons, and for connecting the new knowledge with the previous knowledge (Nindiasari et al., 2014). By carrying out those activities, it is expected for students to motivate to think with having a reason or to think logically.

In any mathematics teaching-learning approach, there are some variables that may be affected on attaining students' good grades on mathematical abilities in general included MLTA, among other things are self-regulated learning (SRL), and mathematical prior knowledge (MPK). The SRL term does not connote learning individually without assistance from other person, but the term constitutes careful self- designing and monitoring processes toward cognitive and affective activities in solving academic tasks. Further, by referring to the nature of mathematics as a systematic discipline, that mathematics contents are composed consecutively and logically, so it is rational that comprehending MPK well, will help students to master the next mathematics contents and the more complex mathematics processes such as MLTA as well. Those aforementioned arguments have motivated researchers to conduct a study for examining the role of MTL and MPK toward the attainment and the normalized gain of students' MLTA and SRL.

RESEARCH QUESTIONS

The research questions are as follows:

1. Are there any differences in the achievement and improvement of MLTA and SRL between the students who received MTL and the students who received conventional learning, both observed overall and based on MPK?
2. Are there any interactions between learning approach and MPK to the achievement of MLTA and SRL?
3. What kinds of difficulties are faced by the students in solving the problems of using mathematical logical thinking?

RESEARCH OBJECTIVES

Based on the research question above, this study aims as follows:

1. To review the differences in achievement and improvement of MLTA and SRL between the students who received MTL and the students who received conventional learning both reviewed overall and based on MPK.

2. To examine whether there is any interaction between learning approach and MPK to the achievement of MLTA and SRL.

3. To describe the difficulties which have been experienced by the students in solving the problems of mathematical logical thinking.

CONTEXT AND REVIEW OF LITERATURE

Mathematical Logical Thinking Ability

In general, logical thinking is defined as a process of making judgments, reasons, and other forms of dynamic thinking to arrive at correct conclusion. Some of experts analyze the term of logical thinking differently. Tobin and Capie (1981) by referring to Piaget's theory of child intellectual cognitivedevelopment, assessed logical thinking ability of students by using the Test of Logical Thinking (TOLT) which enclosed five components namely: controlling variable,proportional reasoning, probabilistic reasoning, correlational reasoning, and combinatorial reasoning. Some of other experts define the term of logical thinking as follow: to draw conclusion by using reasoning consistently (Albrecht, 1984); thinking based on reasoning (Strydom, as cited in Aminah, 2011); thinking based on certain pattern or based on the rules of inference (Suryasumantri, Minderovic&Sponias, as cited in Aminah, 2011); thinking which included induction, deduction, analysis, and synthesis activities (Ioveureyes, as cited in Aminah, 2011). When the object of logical thinking is mathematical problems or ideas, then it is called mathematical logical thinking. Macdonald (Mubark, 2005) described the logical thinking in mathematics as "the idea that there are certain basic rules of grammar with which we can organize our discussion in mathematics is what make it possible to establish that certain things are 'true' in mathematics".

Next, some researchers developed tests which are specifically designed to measure logical thinking ability in the field of study or particular topics constructed by Piaget's logical operation. Norman (Sezen& Bulbul, 2011) measured the logical thinking ability for the chemistry topic, which components consist of proportional reasoning, variant control, probability reasoning, relation, and association. Leongson&Limjap (2003) assessed MLTA of high school students. The reasoning pattern measured covers classification,seriation, logical multiplication,compensation, proportionality, probability,and correlation. The items are limited to geometry, arithmetic, statistics and algebra. Sumarmo, et al., (2012) researched the MLTA of high school students on proportional reasoning, probability reasoning, correlational reasoning, combinatorial reasoning, analogy, and mathematical proof.

In this research, the MLTA components measured are as follows.

1) Proportional reasoning.

Proportionality refers to relative magnitude of the increase and decrease of ratios. Proportional thinking is the establishment of relations of one part to another or of a whole with respect to magnitude, quantity or degree (Leongson&Limjap, 2003).

2) Probability reasoning

Probability refers to reasons in time of the likelihood of possible outcomes. Probability is the chance of an event. Probability reasoning is ability to count the number of all objects (N) and the number of a certain object (n) among them, and determine the chance of selection as a fraction (n/N) (Leongson&Limjap, 2003).

3) Correlational reasoning

Leongson&Limjap (2003, p. 7) defined "Correlational thinking is the establishment of correlation or causal relationship. It may also refer to the presentation or setting forth so as to show relationships." Also, described correlational reasoning as "Can reason with relationships of variables or symbols".

4) Combinatorial reasoning

Bernoulli (Batanero et al., 1997) described combinatorics as "the art of enumerating all the possible ways in which a given number of objects may be mixed and combined so as to be sure of not missing any possible result." It can be said that combinatorial reasoning is the ability to combine different variables of a set containing those variables to make all possible combinations.

5) Generalization

Mason et al. (Mubark, 2005, p. 7) defined the process of generalization as "moving from a few instances to making guesses about a wide class of cases." Stacey (Mubark, 2005, p. 7) described generalization as the process whereby "general rules are discovered by articulating the patterns observed in many particular cases." Meanwhile, according to Tall (1991) the term "generalization" is used in mathematics to denote process in which concepts are seen in broader context and also the product of that process.

6) Analogy

The definition of analogy can be understood from the definition of mathematical analogical thinking expressed by Kinard&Kozulin (2008, p. 88) as follows

Analyzing the structure of both a well-understood and a new mathematical operation, principle, or problem, forming relational aspects of the components of each structure separately, mapping the set of relationships from the well-understood structure to the set of relationships for the new structure, and using one's knowledge about the well-understood situation along with the mapping to construct understanding and insight about the new situation.

7) Mathematical proof

Milton & Reeves (Mubark, 2005) described mathematical proof as that which includes "the formation of a chain of 'valid' reasoning that leads to a conclusion. It is a process of 'authentication' or a process wherein the truth or fallacy of a claim is established."

Self-Regulated Learning

Observing the processes which occur on MLTA, so MLTA is classified as higher-order mathematical thinking that need specific affective behaviors. One of affective behaviors is self-regulated learning (SRL). There are some other terms related to SRL, namely self-regulated thinking, self-directed learning, self-efficacy, and self-esteem. The meanings of those terms are not same exactly but they have some similar traits, and they have positive role on learning. Among those terms, the most important role to learning process is SRL.

Some of writers (Zimmerman, 1990;Pintrich, 2000; Schunk&Ertmer,2000; Paris & Paris, 2001;Wongsri et al., 2002) defined SRL in different ways but they had three main similar characteristics namely: planning a goal, selecting strategy, and monitoring cognitive and affective process happened in solving an academic task. SRL constitutes careful self-designing and monitoring process toward cognitive and affective processes in solving academic tasks. Theyproposed that SRL constitutes recursive cycle of cognitive activities that enclosed: to analyze tasks, to select, to adopt, or to determine approach strategy for obtaining learning goal; and to monitor the learning outcome and strategy had been done. Based on above experts' ideas then Sumarmo (2013) summarized the indicator of SRL as follows: a) to have intrinsic learning initiative and motivation; b) to have habit of diagnosing learning needs; c) to determine learning goal/target; d) to monitor, to manage, and to control learning; e) to consider difficulties as challenges; f) to take advantage of and to seek relevant sources; g) to select and to implement learning strategy; h) to assess learning process and learning outcome; i) to possess self-efficacy and self-concept.

Briefly, self-regulated learning is an individual activity to regulate his/her own learning activities that involves controlling and monitoring aspects (planning goals, managing, monitoring, and evaluating learning processes and outcomes), aspects of motivation (interest, effort, persistence, self-efficacy), and aspects of behavior (utilizing the environment to optimize learning).

Metacognitive Teaching-Learning Approach

Metacognitive teaching-learning approach (MTLA) constitutes an approach which emphasize on promoting students' awareness toward their abilities through accustomization to pose metacognitive questions or problems that enclose understanding on mathematics concepts and problems; to improve connection among new knowledge and previous knowledge; to use relevant strategy; and to reflect process and outcome. (Nindiasari et al.,2014).Metacognitive strategy instruction familiarizes students with five factors of metacognitive knowledge including planning, evaluation, problem solving, directed attention, mental translation and person knowledge; and these factors can develop self-regulated learning and provide a context for interpretation (Rahimirad&Zare-ee, 2015).

In this research, the MTLA contain four main components namely: think-aloud; metacognitive scaffolding, pair discussion, group discussion, class discussion, and metacognitive journal writing. First component of MTLA is think-aloud. The think-aloud strategy begins with modeling by the teacher, expressing loudly all thoughts and feelings that arise while carrying out a task; for example when solving problems, conducting investigations, reading textbook notes, and so on. So, students can hear it and follow the demonstrated thought process.

The second component of MTLA is metacognitive scaffolding. It constitutes an effective strategy to enter the ZPD on Vygotsky's theory (Kinard&Kozulin, 2008). It bridges the gap between what students can do on their own ways and what they can do with assistance by others.In scaffolding, through posing questions teachers provide opportunities to students to expand their previous skills and knowledge. So, by scaffolding, teachers transform complex and difficult tasks become easier tasks to handle and to manage by students.

The third component of MTLA is pair discussion, group discussion, class discussion. According toVygotsky's theory of social constructivism, learning meaningfully will occurs in a social context. When students interact with each other, they share information and suggestion to other members of the group. All members of the group believe that they need each other and receive feedback and they share their ways of thinking and their ways to solve problems to each other. By metacognitive scaffolding, then students construct their new insight, knowledge, and skills meaningfully. Like that, learning in small groups will motivate students to be able to overcome conflict and contradiction which arise while discussion happened and they construct a new and more appropriate knowledge.

The fourth component of MTLA is metacognitive journal writing. When students write an interesting topic in order to be published in a journal, they should compile it accurately. For this reason, students should clarify and reflect their thought rationally and precisely. These activities need students use their metacognitive thinking.

Relevant Studies

Some studies reported the superiority of metacognitive teaching-learning than conventional teaching on enhancing various mathematics abilities such as: Muin (2005) on high level mathematical thinking ability andNindiasari et al. (2014) on mathematical understanding and communication abilities and on mathematical reflective thinking ability. All three of them are studies toward senior high school students. Besides that, some studies on MLTA(Sumarmo, 1987;Pape, et al., 2003;Yenilmez et al., 2005; Sumarmo et al., 2012; Rohaetiet al, 2013;Setiawati, 2014) toward high school students reported that MLTA was difficult mathematics task for the majority of high school students. That statement was portrayed in the findings of the studies that students attained MLTA at low to medium grades. Maya and Sumarmo (2011) by using direct and indirect teaching approach reported different finding on students' mathematical reasoning ability which it was categorized at good grade.

Some studies concerning SRL of high school and tertiary students (Fahinu, 2007; Ratnaningsih, 2007;Sugandi, 2010;Tandililing, 2010; Qohar&Sumarmo, 2013; Rohaeti et al., 2013; Jayadipura, 2014) reported similar findings among others, various innovative teaching approaches confer good role toward students' SRL. Students who were taught by various innovative teaching approaches attained higher grade SRLthan students who were taught by conventional teaching. Those findings pointed out that various innovative teaching-learning which confer opportunity to students for learning actively encouraged students' SRL. Besides that, other variable such as PMK gave positive role to development of students' SRL (Qohar&Sumarmo, 2013, Ratnaningsih, 2007, Sugandi, 2010). Those studies reported the higher students' PMK and school level it were found the higher students' SRL as well. Likewise, Yang (Sumarmo, 2013) reported positive finding that students with high SRL: tended to learn better in their own control, are able to control, to evaluate, and to manage their learning effectively, to save their time in solving their tasks, and to manage their time efficiently.

METHOD

Experimental Design

The experiment design which was used in this research is *Nonequivalent [Pretest and post-test] Control-Group Design* as follows:

Group A	O———	X———	O
Group B	O		O

In such design, X = Metacognitive teaching-learning, O = Test of MLTA and SRL, the group A is an experimental class and the group B is a controlling class.

Participants

Subject of the research was the 10th grade students of a state senior high school at West Java Province, Indonesia. The experiment class consists of 36 students, 15 male-students and 21 female-students;and the control class consists of 34 students,12 male-students and 22 female-students. The implementation and the data collection at school were held during one full-semester. The materials which were discussed during conducting the research were (1) forms of exponents, roots, and logarithms, (2) quadratic functions and parabola, (3) quadratic equations and quadratic inequalities, and (4) system of linear equations.

Instruments

All of the instruments were developed by the researchersin this study and through doing the try outto fulfill the requirements of qualified validity, reliability.

1) Test of Mathematical Prior Knowledge (MPK)

Test of MPK was required to measure the students' mathematical prior knowledge about the materials of mathematics which were studied before, when they were at junior high schools. The materials support in learning the core of discussion which was discussed during this research. Type of MPK test items was short complete, all of it was 20 items. The right answer was given score 1, and the wrong answer was given score 0. The ideal maximum score was 20. The category of MPK was as follows.

Table 1
Category of MPK

Group	Mastery Level	Scores
High	75% – 100%	15 – 20
Middle	55% – 74%	11 – 14
Low	< 55%	0 – 10

The validity coefficients ofthose items ranged from 0.42 to 0.66 (medium and high), and thereliability coefficient was 0.84 (high).

2) Test of Mathematical Logical Thinking Ability (MLTA)

Mathematical logical thinking ability was measured by the test for mathematical logical thinking. The test consisted of eight items, which belong to seven main aspects as described above. The scoring system of mathematical logical thinking was suited from complexity and accomplishment of every question.Three questions each had a score of 1 to 4, one question had a score of 1 to 6, and four questions each had a score of 0 to 8. So, sum of ideal maximum score was 50.The validity coefficients (r_{xy}) ofthose items ranged from 0.50to 0.82 (medium and high), and the reliability coefficient (r_{11}) was 0.72 (high).

3) Self-Regulated Learning (SRL) Scale

Students' SRL behavior was measured using Likert model attitude scale. The scale contained 42 items of statements, with aspects revealed: (1) controlling and monitoring (planning goals, managing, monitoring, and evaluating learning processes and outcomes), (2) motivation (interest, effort, persistence, self-efficacy), and (3) behavior (utilizing the environment to optimize learning).The discriminatory power coefficients (T) of the items ranged from 1.85 to6.77(t_{tab} = 1.76), and thereliability coefficientwas 0.90 (high).

Techniques of Data Analysis

The data which were processed was the scores of pre-test, post-test, and N-gain. To the collected data, the analysis of inferential and descriptive statistics was done. The statistic analysis used in this research is one way Anova, *t*-test, Mann-Whitney test, danKruskal-Wallis testwith significance level 0.05.

FINDINGS

1) Mathematical Logical Thinking Ability (MLTA) and Self-Regulated Learning (SRL)

The description of MLTA and SRL of students are presented in Table 2.

Table 2

Pre-test and Post-test and the N-Gain of MLTA and SRL

Variable	MPK	Stat	Metacognitive Teaching-Learning (MTL)				Conventional Teaching			
			Pretest	Post-test	N-gain	n	Pre- test	Post- test	N-gain	n
MLTA	High	$\bar{x}$	6.88	41.13	0.79	8	8.10	37.50	0.70	10
		%	13.76	82.26			16.20	75.00		
		s	4.02	2.75	0.06		2.23	2.23	0.09	
	Med	$\bar{x}$	2.07	32.21	0.63	14	3.36	27.27	0.51	11
		%	4.14	64.42			6.72	54.54		
		s	2.13	6.29	0.12		2.91	5.33	0.12	
	Low	$\bar{x}$	1.36	21.36	0.41	14	1.69	20.62	0.39	13
		%	2.72	42.72			3.38	41.24		
		s	2.47	4.38	0.08		1.55	4.33	0.08	
	Total	$\bar{x}$	2.86	29.97	0.58	36	4.12	27.74	0.51	34
		%	5.72	59.94			8.24	55.48		
		s	3.47	9.14	0.18		3.48	8.31	0.16	
SRL	High	$\bar{x}$	144.38	157.13	0.18	8	152,00	152,50	0.01	10
		%	68.10	74.12			71.70	71.93		
		s	15.22	11.09	0,15		4.78	12.48	0.16	
	Med	$\bar{x}$	140.14	151.43	0,14	14	136.09	133.82	−0,04	11
		%	66.10	71.43			64.19	63.12		
		s	14.11	10.55	0.14		11.60	11.20	0.13	
	Low	$\bar{x}$	131.71	140.64	0.11	14	142.46	144.00	0.02	13
		%	62.13	66.34			67.20	67.92		
		s	9.56	14.32	0.14		8.84	11.72	0.10	
	Total	$\bar{x}$	137.81	148.50	0.14	36	143.21	143.21	0.00	34
		%	65.00	70.05			67.55	67.55		
		s	13.46	13.68	0.14		10.77	13.65	0.13	

Note: % out of Ideal score; Ideal score of MLTA is 50; and Ideal score of SRL is 212.

The findings points out that in entirely students, in pre-test there is no difference grades of MLTA of students in both classes and those grades are at very low level (5.72% and 8.24% out of ideal score). Like that, in post-test there is no difference of MLTA and its N gain, between students who were taught by MTL (59.94 % out of ideal score, N-gain was 0,58) and students who were taught by conventional teaching (55.48 % out of ideal score, and N-gain was 0,51), and those students' grades of MLTA are at medium grade level.

One of the difference tests results of MLTA are presented in Table 3.a, Table 3.b, and Table 3.c, and othersare summarized in the Table4.

Table 3.a
Tests of Normality of MLTA Post-Test on Total Students

Tests of Normality							
	Teaching	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Post-test_ MLTA_Total	Metacognitive	.144	36	.058	.927	36	.090
	Conventional	.144	34	.071	.953	34	.051

a. Lilliefors Significance Correction.

The data in the two classes were normally distributed (Sig = 0.090> 0.05 and Sig = 0.051> 0.05). Furthermore, the homogeneity of variance test was done.

Table 3.b

Tests of Homogeneity of Variances of MLTA Post-Test on Total Students

Test of Homogeneity of Variances			
Post-test_MLTA_Total			
Levene Statistic	df1	df2	Sig.
.248	1	68	.620

The data in both classes had a homogeneous variance (Sig = 0.620 > 0.05). The final step was to test the difference between two means.

Table 3.c
Tests of Difference between Two Means of MLTA Post-Test on Total Students

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Post-test_MLTA_Total	Equal variances assumed	.248	.620	.914	68	.364	1.24265	1.35891	-1.4690	3.9543
	Equal variances not assumed			.916	67.99	.363	1.24265	1.35625	-1.4637	3.9490

There is no difference in the MLTA post-test of the experimental class and control class students (Sig = 0.364> 0.05).

Table 4
Difference Tests of MLTAInter Groups on the Same MPK Level

MPK	Pretest		Post-test		N-Gain	
	Sig.	Interpretation	Sig.	Interpretation	Sig.	Interpretation
High-High	0.446	No difference	0.048	Different	0.033	Different
Med-Med	0.278	No difference	0.041	Different	0.023	Different
Low-Low	0.154	No difference	0.661	No difference	0.543	No difference
Total-Total	0.074	No difference	0.364	No difference	0.190	No difference

Look out over from level of MPK (high, medium, low), in both classes, the higher the students' MPK there are found the higher students' MLTA and their N-gain of as well or students' MLTA and their N-gain are increased (see Table 2). These findings points out that MPK take a good role toward the attainment of students' MLTA and their N-gain.

Table 5 presents one of thedifference tests results of students' SRL on high level, and others are summarized in the Table 6.

Table 5
Tests of Difference between Two Means of MLTA N-gain of High Level Students

Ranks

	Teaching	N	Mean Rank	Sum of Ranks
N.gain_SRL_H	Metakognitif	8	12.44	99.50
	Konvensional	10	7.15	71.50
	Total	18		

Test Statistics[a]

	N.gain_SRL_H
Mann-Whitney U	16.500
Wilcoxon W	71.500
Z	-2.090
Asymp. Sig. (2-tailed)	.037
Exact Sig. [2*(1-tailed Sig.)]	.034[b]

a. Grouping Variable: Pembelajaran
b. Not corrected for ties.

There is a difference in the increase of the students' SRL on high level in the experimental class and control class (Sig. = 0.037 <0.05).

Table 6
Difference tests of SRL inter groups on the same MPK level

MPK	Prescale		Post-scale		N-Gain	
	Sig.	Interpretation	Sig.	Interpretation	Sig.	Interpretation
High-High	0.350	No difference	0.419	No difference	0.037	Different
Med-Med	0.427	No difference	0.001	Different	0.003	Different
Low-Low	0.011	Difference	0.543	No difference	0.080	No difference
Total-Total	0.069	No difference	0.036	Different	0.000	Different

In contrast to MLTA, the findings of SRL in Table 2 point out that in entirely students, there is difference grades of SRL of students in both classes and those grades were at fairly good grades (0.14% and 0.00% out of ideal score). Table 3 shows that on students of medium MPK, the attainment of SRL of students who were taught by MTL were better than students who were taught by conventional teaching

2) Interaction between Learning Approaches and MPK to MLTA

Further analysis, by using tests of between-subjects effects, the study found that there was no interaction between teaching approaches and level of MPK toward students' MLTA, but there has been an interaction between teaching approaches and level of MPK toward students' SRL (Table 7 and Table 8).

Table 7
Tests of Between-Subjects Effects

Dependent Variable: MLTA_ Post-test					
Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	3806.881[a]	5	761.376	32.920	.000
Intercept	60539.647	1	60539.647	2617.568	.000
Learning	161.744	1	161.744	6.993	.010
MPK	3632.809	2	1816.405	78.536	.000
Learning * MPK	59.319	2	29.660	1.282	.284
Error	1480.205	64	23.128		
Total	63694.000	70			
Corrected Total	5287.086	69			

a. R Squared = .720 (Adjusted R Squared = .698)

Table 7 shows that there is an influence of the learning approach to MLTA achievement(Sig. = 0.010 < 0.05). As previously tested, MLTA achievement differs between experimental class and control class occurs in high and middle level students. Besides that, there is also an influence of the students' MPK toward the MLTA achievement(Sig. = 0.000 < 0.05). Noted that, there is no interaction between learning-teaching approaches and MPK toward MLTA (Sig. = 0.284 > 0.05).

Table 8
Tests of Between-Subjects Effects

Dependent Variable: SRL_Post-Scale					
Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	3910.989[a]	5	782.198	5.396	.000
Intercept	1444019.760	1	1444019.760	9961.275	.000
Learning	665.289	1	665.289	4.589	.036
MPK	2016.886	2	1008.443	6.957	.002
Learning * MPK	1428.657	2	714.328	4.928	.010
Error	9277.654	64	144.963		
Total	1503849.000	70			
Corrected Total	13188.643	69			

a. R Squared = .297 (Adjusted R Squared = .242)

From Table 8, it is found that there is an influence of the learning approach to SRL attainment (Sig. = 0.036 < 0.05), beside there is an influence of students' MPK to SRL achievement. Also there is an interaction between the teaching approaches and the MPK toward the SRL (Sig. = 0.010 < 0.05).

The interaction patterns are shown in Figure 1 and Figure 2.

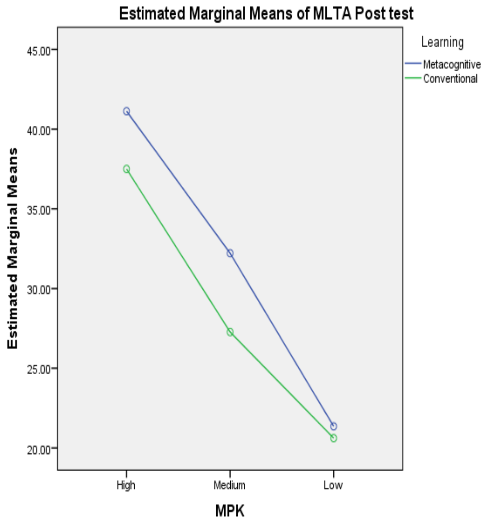

Figure 1
NoInteraction between learning approaches and MPK toward MLTA

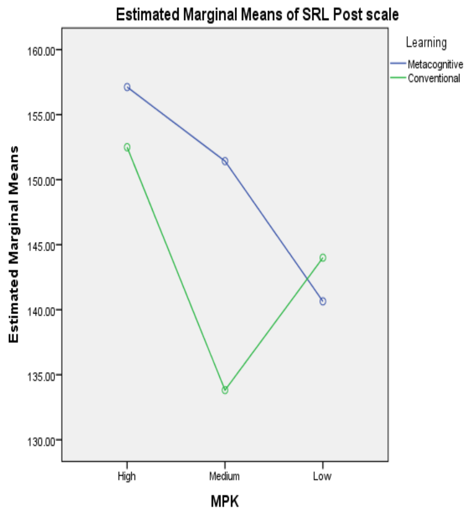

Figure 2
There is an interaction between Learning Approaches and MPK toward SRL

According to Figure 1, both in the metacognitive and conventional classes, the achievement of the highest MLTA is achieved by the groups of students with high MPK, followed by the groups of students with medium MPK, and the lowest has been achieved by the groups of students with low MPK. In contrast to the achievement of MLTA, metacognitive learning does not alter the sequence of achievement of students' SRL, whereas conventional learning results the medium group being under the low group in behavior of SRL (Figure 2).

3) Students' Difficulties on MLTA Tasks
Students' post-test grades in each item of MLTA in both classes of teaching approaches are presented in Table 9.

Table 9
Students' grade of each item of MLTA on post-test

Indicator	Ideal Score	MTL Class			Conventional Class		
		$\bar{x}$	%	Rank	$\bar{x}$	%	Rank
1. Proportional reasoning	4	3.33	83.33	1	3.32	83.09	1
2. Probabilistic reasoning	8	4.11	51.39	5	3.65	45.59	5
3. Correlational reasoning	4	2.42	60.42	4	2.26	56.62	4
4. Combinatorial reasoning	8	5.39	67.36	3	5.24	65.44	3
5. Generalization	8	3.61	45.14	6	3.21	40.07	6
6. Analogy	12	9.03	75.23	2	8.32	69.36	2
7. Mathematical proof	6	2.08	34.72	7	1.74	28.92	7
Mean			59.94			55.48	

On pretest, all students on both teaching approaches (MTL and conventional) realized difficulties in solving each item of MLTA (Table 9), and they were at very low grades, even there were no students can solve test item on mathematical proof. It was rational and understandable as students had not been taught the mathematics contents and processes. However, on post-test, the students who were taught by MTL and by conventional teaching still posed difficulties on solving MLTA on generalization, and mathematical proof.

DISCUSSION

The results of the present study are different with findings of other studies that on MLTA and its normalized gain of students of experiment class (taught by innovative teaching) were better than that of students who were taught by conventional teaching (Sumarmoet al., 2012; Rohaeti et al., 2013; Setiawati, 2014). However, considering the quality of students' MLTA and its normalized gain, the findings of this study are similar to the findings of other studies namely at medium grade level (Sumarmo, 1987; Sumarmo, et al., 2012; Rohaeti, et al., 2013; Setiawati, 2014).

This finding is similar to the findings of other studies that MPK take a role to attainment of various mathematics abilities (Tandililing, 2010; Qohar and Sumarmo, 2013; Setiawati, 2014; Kurniawati, et al., 2014). This finding is in line with the nature of mathematics as a systematic discipline namely: mastering previous mathematics concepts well will help to attain better grades on the next mathematics contents and processes. Table 2 points out that on students of high and medium MPK, the attainment of MLTA and its N-gain of students who were taught by MTL are better than students taught who were by conventional teaching. However, there is no difference of MLTA and their N-gain of students of low MPK and of total.

On students of high and medium MPK, the N-gain of SRL of students who were taught by MTL is better than students who were taught by conventional teaching. The finding of SRL in this study is different with the findings of other studies (Rohaeti, et al., 2013) that there was no difference on SRL between studentstaught by innovative teaching and taught by conventional teaching. This finding is similar to the findings of other studies that SRL of experimental group students attained better grade of SRL than that of students who were taught by conventional teaching (Fahinu, 2007; Ratnaningsih, 2007; Qohar and Sumarmo, 2013).

The result provides an illustration that the two lessons did not change the sequence of students' MLTA. It also appears that in each pair of equal groups, the group of students of the metacognitive class is higher than the conventional class. In accordance with the previous test results, although the students' scores in the metacognitive class are higher, the difference is not significant, whereas the MPK differs significantly. This shows the absence of interaction between learning with MPK in the achievement of MLTA.

For students in both classes, the most difficult item was about mathematical proof. This finding is in line with findings of some studies on tertiary students (Maya &Sumarmo, 2011; Yerizon, 2011) and similar to the results of a survey conducted in Indonesia in 1999 (Suryadi, 2012). It is reported that math activities that are deemed difficult by students to learn and by teachers to teach them include justification or proofing, problem solving which requires mathematical reasoning, finding generalizations or conjectures, and finding relationships between data or facts given. Furthermore, the difficulty of mathematical proofing is a common symptom, because even in college, as revealed by Moore (Wahyudin, 2012), proofing is an area that is very difficult for students. This difficulty of mathematical proofing can be caused by students having difficulty using and exploring information, extracting hidden facts, seeing relation to other concepts, making conjectures, making assumptions and investigating the consequences, or justifying results.

CONCLUSION

Based on the findings and discussion, the study draws some conclusions as follows. In entirely students, there are no difference grades of mathematical logical thinking ability and its normalized-gain between students who were taught by metacognitive teaching-learning and students who were taught by conventional learning. Nevertheless, on students with high and medium of mathematical prior knowledge, on mathematical logical thinking ability and its normalized gain, students who were taught by metacognitive teaching-learning got higher grade than students who were taught by conventional teaching. Like that, in entirely students and in each level of mathematics prior knowledge (high, medium, and low), there are different grades of self-regulated learning between students who were taught by metacognitive teaching-learning and students who were taught by conventional learning. The students' SRL is at fairly good grades.

There is no interaction between learning approaches and mathematical prior knowledge on the students' mathematical logical thinking ability, and there is an interaction between learning approaches and mathematical prior knowledge on self-regulated learning. There is no single or certain mathematics teaching approach which is able to guarantee to attain students' higher grade MLTA. However, with any teaching approach, teacher should accustomed students to pose metacognitive questions to their-selves, and then they attempt to answer the questions or to solve the problems together with relevant reasons. By these continuous activities, students' MLTA is expected to increase gradually.

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